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Design and Analysis of a Pressurized Composite Fuel Tank

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Table of Contents

Abstract.....	1
Background.....	2
Introduction	2
Requirements	3
Fuel tank material trade study.....	3
Table 1: Material Cost Summary [2]	4
Table 2: Material Properties Summary.....	5
Table 3: Material Ratio Summary.....	5
Preliminary Design.....	7
Review of mechanics on pressure vessels.....	7
Review of composite materials.....	8
Composite pressure vessel analysis	9
Background on Failure Criterion	13
Failure Criterion Analysis.....	15
Analysis Results.....	19
Table 4: Pressure Vessel Summary.....	20
Manufacturing Analysis	20
Table 5: Mold Product Summary.....	23
Table 6: Construction Summary Comparison.....	24
Procedure	24
Manufacturing Results.....	27
Conclusion.....	30
Acknowledgements.....	33
References.....	34
Appendix.....	36
A: Calculations for shape and size of actual tank.....	37
B: Review of mechanics on pressure vessels	38
C: Information on Gr33 150 gsm/BT250E-1, 24 graphite/epoxy unitape.....	40
D: Actual optimization of ply orientation and spreadsheets	48
E: Spreadsheet data for Figures 5-7: Principal stresses versus number of plies.....	58
F: Material information on composites and CARE MOLD™	60
G: Composite roll measurements.....	67
H: Dimensional analysis on testing pressure	69

Abstract

An investigation works towards obtaining a lightweight, cost-effective pressurized fuel tank for a hybrid sounding rocket with a capacity of 20 kg of either Nitrous Oxide or Hydrogen Peroxide. Trade studies result in analyzing a graphite epoxy prepreg composite tank with unidirectional lamina designed for a burst pressure of 2000 psi. The analysis resulted in a pressure vessel in the shape of a cylinder with spherical ends. The tank's diameter is 8 inches and the total length is 5 feet. The analysis further determined the fuel tank's layered construction of 6 plies oriented 0 degrees and 90 degrees with 10 plies oriented at 45 degrees. This gave a total of 22 plies for a thickness of 0.11 inches with a tank factor of 18785 and a mass of 3.7 kg. Hand lay-up manufacturing techniques are discussed for composite fuel tanks as well as mold and tooling comparisons.

Background

There are four basic categories for structural materials. These include metals, polymers, ceramics, and composites. Composites are defined as two or more separate materials combined on a macroscopic level.¹ Composite materials are starting to play an important role for various structures, pressure vessels being no exception. Composites tend to be lighter than metals, but very strong which makes this material appealing for any application where weight constraints are imposed.

Pressurized fuel tanks are an integral part of current rocket motors such as liquid or hybrid systems. Most are made from either a metal or fiber composite and vary in size, weight, and operating pressure. By their very nature, rockets have strict weight requirements placed on them in order to overcome Earth's gravity. Because of these constraints, there is a need for lightweight components like pressurized fuel tanks.

At the United States Air Force Academy, people are in the process of building a hybrid sounding rocket. Their objective is to prove that hybrid rockets are worth while for space application. Current pressure vessels available for fuel tanks are not appropriate for this particular hybrid rocket design. Most of them are too heavy, and their operating pressure is too high for its application. This even holds true for available composite pressure vessels! Therefore, a customized, lightweight, pressurized, fuel tank is needed for this hybrid sounding rocket.

Introduction

The following research steps through one of many solutions to achieve a pressurized fuel tank for this rocket system. Trade studies help answer questions like: What material is

appropriate? How should this vessel be made? How much is it going to cost? Note that this whole research is focusing on one pressure vessel for a fuel tank for a hybrid sounding rocket, not any sort of mass production.

This paper reviews the requirements that must be met by the fuel tank, which help guide the research. Trade studies help select a material and preliminary design analysis helps determine shape and geometry. Options are discussed for manufacturing a pressurized fuel tank, which emphasizes hand lay-ups. Before any analysis starts, understanding requirements is paramount for this research.

Requirements

A fuel tank is needed for a hybrid sounding rocket being designed at the United States Air Force Academy. This fuel tank must be lightweight and have an operating pressure of 1000 psi. This design will incorporate a factor of safety of 2 resulting in a burst pressure of 2000 psi. Furthermore, this fuel tank must be compatible to hold 20 kg of Nitrous Oxide or Hydrogen Peroxide. The hybrid rocket design constrains the fuel tank to an 8-9 in diameter. Ultimately, we would like a cost-effective, pressurized, fuel tank. The question is how to meet these requirements. Trade studies help solve this problem to select a material and manufacturing process suitable for the application.

Fuel tank material trade study

The follow trade study is designed to help select a material for the fuel tank. As mentioned previously, structures can be made from metals, polymers, ceramics, or composite materials. Pure polymers will be ruled out because low strength. Ceramics will also be removed

from the trade study because of the difficulty in manufacturing pressure vessels. Therefore metals and composites will be the focus of this trade study.

There are five basic categories this trade study analyzes. These include cost, weight, strength, stiffness, and manufacturability. Three different ratios help compare the various material properties. These consist of: strength-to-weight, stiffness-to-weight, and strength-to-weight-to-price.

Table 1 shows a cost comparison for three kinds of metals and two kinds of composite materials.

Table 1: Material Cost Summary²

Material	Price \$/ton
Titanium Alloys	10,190-12,720
Stainless Steels	2,400-3,100
Aluminum Alloys	2,000-2,440
Boron/Epoxy Composites	330,000
Carbon-fiber-reinforced-polymers (CFRP)	200,000
Bryte Technologies, Inc. Gr 33 150 gsm/BT250E-1, 24	50,000*

Note that 60% of the cost for Boron/Epoxy composites is materials and 30% of the cost for CFRP is also materials. The remaining percentages account for the cost of fabrication.³ Note that the material costs from Bryte Technologies is current (1998) compared to the rest of Table 1, which are from 1993.

Table 2 outlines the various strength, stiffness, and weight of the materials. This is shown through material properties such as tensile strength, tensile modulus, and density.

Table 2: Material Properties Summary

Material	Tensile Strength (MPa)	Tensile Modulus (MPa)	Density (g/cm ³)
Aluminum (6061 T6)	310	69	2.71
Steel (SAE 4340)	1034	200	7.83
Boron Fibers	3516	400	2.57
Carbon Fibers (P-55)	1724	379	1.99
Gr 33 150 gsm/BT250E-1, 24	1965	13100	1.55

Table 3 summarizes ratios to better compare the various materials. The first ratio compares strength-to-weight, while the second compares stiffness to weight. Using Table 2, tensile strength and density are used for the first ratio, while tensile modulus and density are used for the second ratio. High ratios mean more strength or stiffness for less weight, which make the material more appealing. The last ratio compares price to strength-to-weight. The lower this ratio is the better. The material becomes more attractive if there is a low price tag for the strength-to-weight. This last ratio uses the values for the first ratio along with the average prices found from Table 1.

Table 3: Material Ratio Summary

Material	Strength/Weight (MPa cm ³ /g)	Stiffness/Weight (MPa cm ³ /g)	\$/Ton(Strength/Weight))
Aluminum (6061 T6)	114	25	28
Steel (SAE 4340)	132	26	42
Boron Fibers	1368	156	241
Carbon Fibers (P-55)	866	190	231
Gr 33 150 gsm/BT250E-1, 24	1310	8452	38

Using the previous tables, it is easier to compare different metals to composites. Titanium is less expensive than composites and for a metal, titanium has a low density, with good strength and stiffness. However it is one of the hardest metals, which makes it difficult to machine. This would drive the cost up, especially since only one pressure vessel is needed.

Aluminum also cost less than composites, but for a metal the material properties are nothing spectacular. The one major benefit of aluminum is that it is easy to machine. Steel seems to be in the middle of the titanium and aluminum comparisons. It has relatively good strength and stiffness characteristics, however it has a rather high density. Furthermore, steel is not as easy to machine as aluminum.

Focusing on composites, obviously the major penalty is cost (Table 1). As mentioned earlier, Table 1 price comparisons were from 1993. Since there has been more use and applications for composites, the industry has increased. This reduced the price of not only the fabrication process, but also the material. (This explains why Bryte Technologies composites cost less than similar composite materials from an earlier reference source.)

Composites display very good material properties from Table 2-much better than aluminum or steel. In addition Table 3 shows that if respective prices, strengths, and weights are normalized, the graphite epoxy prepreg composite material from Bryte Technologies is a reasonable choice. (This ratio falls between those of aluminum and steel.)

Furthermore, composite materials make it easy to fabricate structures. Before they are cured, the material acts like cloth, making it simple to bend or wrap around for complex angles and designs. This can reduce the number of parts for a design. This results in less raw materials, fewer fasteners, and less assembly time.⁴ Composite materials may also possess unique thermal or electrical properties. This means that composites can be designed to support not only structural loads, but also thermal and electrical loads.

After reviewing the pro's and con's of all the materials mentioned, composite materials seem to have the best qualities in a material for a pressurized fuel tank.

Preliminary Design

The main design questions that need to be answered is how big should the tank be, and what is its geometry. Pressure vessels have evolved to be either spherical or cylindrical with spherical (or elliptical) ends. Mechanical analysis shows that these shapes do well for pressure vessels because they do not have any sharp corners, which produce stress concentrations and emphasized microscopic flaws. In order to hold 20 kg of Nitrous Oxide, the preliminary design shows the fuel tank will roughly look like Figure 1. Appendix A give the complete calculations for Figure 1.

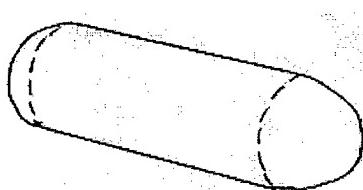


Figure 1: Preliminary geometry for a pressurized fuel tank

The remaining questions are how many layers should the tank be, and how should these plies be oriented? The answers are based on the material properties of the actual composite fibers have.

Review of mechanics on pressure vessels

Since the walls are stiff, and do not bend, the internal forces are tangent to the vessel's surface.⁵ This holds true for thin walled vessels where the research will focus in order to conserve weight. Figures 2 and 3 refer to the biaxial forces common in cylindrical and spherical pressure vessels.⁶ Because the vessel will have a relatively small vessel thickness compared to its radius, the local effects due to radius of curvature can be neglected. This will also allow us to

analyze the tank's surface as a flat laminate. From basic mechanics, the hoop and longitudinal stresses for a cylinder and a sphere may be obtained from Eq. (1) and (2).

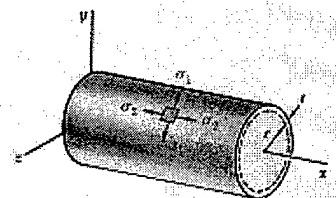


Figure 2: Biaxial stresses across a cylindrical surface

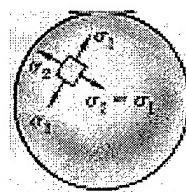


Figure 3: Biaxial stresses across a spherical surface

$$\sigma_h = \frac{pr}{t} \quad (1)$$

$$\sigma_l = \frac{pr}{2t} \quad (2)$$

$\sigma_h = \sigma_1$ = hoop stress
 $\sigma_l = \sigma_2$ = longitudinal stress
 p = pressure in vessel
 r = radius of vessel
 t = thickness of vessel

This means that the longitudinal stress is half of the hoop stress. A detailed review of stress analysis on pressure vessels may be found in Appendix B.

Review of composite materials

As mentioned before, composite materials are defined as two or more separate materials combined on the macroscopic level. This refers to combining lamina (two or more plies) to form an overall structure. For example, combining 20 plies wrapped around a mold could make a *composite hat panel* for an aircraft wing.

There are two components working in every composite ply; the fibers and the matrix. The fibers carry the majority of the load, and are the actual fiberglass or graphite strands. The matrix is the other component of a composite material. It is the resin or epoxy that holds the fibers together, and the plies together. The matrix plays an important part in interlaminar strength, and its lack in strength is usually the main cause for delaminations (separations between plies). However, the matrix plays an important role in transferring and distributing the applied loads to the fibers (i.e. if a shear stress was applied). It also contributes to ductility, toughness, or electrical insulation, and protects the structure from external damage. An important requirement of the matrix is that it must be capable of developing a mechanical or chemical bond with the fibers.⁷

For this research, unidirectional composite material will be analyzed instead of woven composites. Unidirectional composites offer a potential for strong structures since the fibers are straight. Even though woven cloths composites are somewhat immune to delaminations, they sacrifice strength and stiffness since the fibers are slightly bent. Furthermore, thermoset composite matrices will be researched instead of thermoplastic matrices. The latter can be reshaped more than once which could result in limited fuel tank locations, away from the rocket motor.

Composite pressure vessel analysis

The goal here is to find how the plies should be oriented. Should they be at 0° or 90° or somewhere in between? For example, should the fibers run in the longitudinal direction, hoop direction, or a non-trivial direction? In addition, the tank's number of plies must be found so that the tank can operate safely at 1000 psi. For research purposes, the vessel will be analyzed using

Gr 33 150 gsm/BT250E-1, 24 graphite epoxy unitape from Bryte Technologies, Inc. Specific material information may be found in Appendix C. From basic pressure vessel calculations the stresses in the hoop and longitudinal directions may be found. Since the pressure vessel is in a biaxial state of stress, it is important that some plies are oriented along these axis (the fibers run in the direction of stress.) If the plies' fibers line up directly with either axis (oriented at 0° or 90°), the calculations are simple. These calculated hoop and longitudinal stresses may be compared with the actual strength in the composite material, which is given. If the stress exerted on the lamina exceeds how strong the ply really is, it will fail. However, what if the fibers are not oriented along one of the two major axes? The hoop and longitudinal stresses must be converted or transformed into stresses that line up with the fibers. This is referred to transforming applied stresses into principal stresses. Figure 4 shows the convention for fibers oriented in some arbitrary direction.⁸

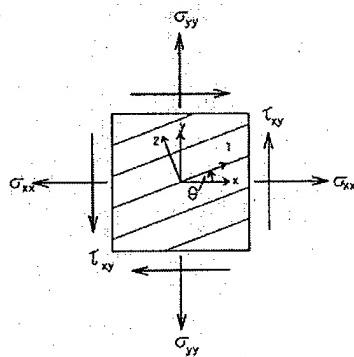


Figure 4: Transformation of stresses

Equations 3, 4, and 5 are the complete transformation equations. For this analysis they enable us to convert longitudinal, hoop, and shear stresses into fiber and matrix stresses.

$$\sigma_2 = \sigma_{xx} \sin^2 \theta + \sigma_{yy} \cos^2 \theta - 2\tau_{xy} \sin \theta \cos \theta \quad (3)$$

$$\sigma_1 = \sigma_{xx} \cos^2 \theta + \sigma_{yy} \sin^2 \theta + 2\tau_{xy} \sin \theta \cos \theta \quad (4)$$

$$\tau_{12} = -\sigma_{xx} \sin \theta \cos \theta + \sigma_{yy} \sin \theta \cos \theta + \tau_{xy} (\cos^2 \theta - \sin^2 \theta) \quad (5)$$

σ_{xx} = longitudinal stress, σ_l

σ_{yy} = hoop stress, σ_h

τ_{xy} = shear stress in xy coordinate frame

σ_1 = fiber stress

σ_2 = matrix stress

τ_{12} = shear stress in fiber and matrix coordinate frame

θ = orientation angle of lamina

The idea here is to find an optimum θ . Note that the optimal fiber direction does not necessarily have to be in the direction of the principal stresses.⁹ The optimal fiber direction may be at some angle to the principal direction. It will depend on the ratio of shear strength to traverse tensile strength. However, since we are analyzing a pressure vessel with more than one ply, this avenue will not be pursued.

The goal of the optimum orientation angle is to reduce the principal shear stress, τ_{12} , to zero. Since yielding is usually controlled by shear, it is important to find this orientation angle. Since σ_x and σ_y are known from the pressure vessel requirements (hoop and longitudinal stresses), Eqs. (3) and (4) can be manipulated to get:

$$2 = \frac{\sigma_1 \sin^2 \theta + \sigma_2 \cos^2 \theta}{\sigma_1 \cos^2 \theta + \sigma_2 \sin^2 \theta} \quad (6)$$

This means that $\tau_{xy} = 0$ which is a safe assumption because of biaxial loads. Solving for θ will give the optimum winding angle for the cylindrical part of the pressure vessel. However, σ_1 and

σ_2 are unknown at this time. If we assume that $\sigma_1 \gg \sigma_2$, then Eq. (6) reduces even further. This results in an optimum winding angle of approximately $= 55^\circ$.¹⁰ With this angle, the shear force in one lamina is balanced by the opposing shear force in the adjacent lamina, therefore producing a net shear force of zero.

However, what if we cannot assume $\sigma_1 \gg \sigma_2$? What if σ_1 is approximately σ_2 ?

Appendix D shows a detailed analysis which incorporates a spread sheet calculation since there were four simultaneous equations to solve. This was done by Gauss reduction and iteration. This resulted in an optimum orientation angle of approximately $\theta = 45^\circ$. This is intuitively correct since $\sin(\theta) = \cos(\theta)$ for $\theta = 45^\circ$, which reduces Eqs. (3), (4), and (5) significantly. Now that the optimum orientation angle is found, how many plies are needed?

An optimal composite design will use the minimal amount of material to resist the given loading state. This is based on the failure criterion used. The basic process to determine the number of plies for this particular pressure vessel uses failure criterion and iteration until the failure criterion is met. From Eqs. (1) and (2), the hoop and longitudinal stresses are based on pressure, radius, and thickness of the vessel. However this thickness refers to the *total* pressure vessel thickness. This is equal to the lamina thickness times the number of plies. This means that Eqs. (1) and (2) become:

$$\sigma_h = \frac{pr}{t_t} \quad (7)$$

$$\sigma_l = \frac{pr}{2t_t} \quad (8)$$

t_t = total pressure vessel thickness = nt^*
 t^* = lamina thickness

Background on Failure Criterion

Design analysis of a structure or a component is performed by comparing stresses (or strains) due to applied loads with the allowable strength (or strain capacity) of the material.¹¹ Strength tends to be highly directional for composites; S_L usually refers to the strength in the fiber direction, and S_T is the strength in the transverse (matrix) direction. Many times the transverse strength is much lower than its counterpart. Even S_L and S_T have compressive and tensile strengths associated with it, which are often different values. Therefore proper equations are needed to incorporate the specific material characteristics resulting in correct failure evaluation. With off-axis or multi-axial loading, lamina failure is assumed to be characterized by using failure criterion. The goal is to quickly estimate lamina failure under non-trivial or complex loading conditions.¹²

Exceeding the transverse strength usually causes ply failure. This motivates the design to not only have the proper number of plies, but also the proper ply orientation. If designed correctly, the transverse loading in one ply will be partially carried by the fibers in another. How much is carried depends on the orientation angle.

Failure analysis assumes no micro-mechanical failures. This means no fiber pullout, fiber micro buckling, matrix cracking, delaminations, or defects. The application of any failure criterion is to first transform calculated stresses into principal material stresses.

Von Mises failure theory is often used for the prediction of yielding in ductile metals. It is based on principal stress differences and corresponding shear stresses and strains, which drive slip and dislocation in metallic crystals.¹³ Since we are considering a graphite epoxy composite design, Von Mises failure criterion will not be used.

Azzi-Tsai-Hill failure theory is satisfied if

$$\frac{\sigma_1^2}{S_L^2} - \frac{\sigma_1\sigma_2}{S_L^2} + \frac{\sigma_2^2}{S_T^2} + \frac{\tau_{12}^2}{S_T^2} = 1 \quad (9)$$

This means that if the left-hand side of Eq. (9) is greater than or equal to 1, then failure will occur in the lamina. This is a more accurate equation since compressive *and* tensile strengths are incorporated. However, the appropriate values of S_L and S_T for each quadrant of stress space must be used. (i.e. If σ_1 is negative, use $S_{L(+)}$ and if σ_2 is positive, use $S_{L(-)}$. The signs refer to the tension and compressive strengths in the longitudinal direction respectively.)

The Tsai-Wu failure theory predicts failure in an orthotropic lamina under plane stress conditions if Eq. (10) is satisfied (if the left-hand side is greater than or equal to the right-hand side.)

$$F_{11}\sigma_1^2 + F_{22}\sigma_2^2 + F_{66}\tau_{12}^2 + F_1\sigma_1 + F_2\sigma_2 + 2F_{12}\sigma_1\sigma_2 = 1 \quad (10)$$

$$F_{11} = \frac{1}{S_L^{(+)}} \quad F_1 = \frac{1}{S_L^{(+)}} - \frac{1}{S_L^{(-)}} \\ F_{22} = \frac{1}{S_T^{(+)}} \quad F_2 = \frac{1}{S_T^{(+)}} - \frac{1}{S_T^{(-)}}$$

$$F_{11} = \frac{1}{S_{LT}^2} \quad F_{12} = -\frac{\sqrt{F_{11}F_{12}}}{2}$$

$S_L^{(+)}$ = Longitudinal strength in tension

$S_L^{(-)}$ = Longitudinal strength in compression

$S_T^{(+)}$ = Transverse strength in tension

$S_T^{(-)}$ = Transverse strength in compression

S_{LT} = In-plane shear strength

Tsai-Wu failure theory is similar to the Azzi-Tsai-Hill theory, but the coefficients are different. This is just another way of trying to describe when failure will occur among lamina.

The most basic failure theory is the *Maximum Stress theory*. This states that failure will occur if the actual stress is equal to or greater than the corresponding ultimate strength. Equations 11-13 summarize the Maximum Stress Theory.

$$-S_L^{(-)} < \sigma_1 < S_L^{(+)} \quad (11)$$

$$-S_T^{(-)} < \sigma_2 < S_T^{(+)} \quad (12)$$

$$|\tau_{12}| < S_{LT} \quad (13)$$

Using these equations, we are able to estimate when failure will occur in a lamina and expand on this data to predict when the pressure vessel will fail. Since we do not want it to fail before 2000 psi, we can increase the number of layers, n, in Eqs. (7) and (8) until at least one of the failure criterions is met.

Failure Criterion Analysis

One obstacle for the graphite epoxy material from Bryte Technologies Inc. was that transverse strength properties, $S_T^{(+)}$, were not given. In order to predict transverse tensile strength in a unidirectional lamina, I was forced to estimate $S_T^{(+)}$ based on similar values in other composite materials. I used linear interpolation from $S_T^{(+)}$ values of E-glass epoxy and Boron-epoxy to get in the ballpark for graphite epoxy prepreg unidirectional tape. This results in a much lower transverse strength compared to the longitudinal strength. Therefore it is more appropriate to use either Azzi-Tsai-Hill or Tsai-Wu failure theory since these incorporate different longitudinal and transverse strengths.

Using the various failure theories, one may obtain the number of plies needed for the pressure vessel. This is done by first guessing the number of plies. Solving Eqs. (7) and (8), then using these values in Eqs. (3)-(5) results in σ_1 , σ_2 , τ_{12} . Using these values and Eqs. (9)-(13),

it may be determined if any of the failure criterions is met or exceeded. If the latter holds true, we must increase the number of plies until the total stress in the plies is lower than the lamina's strength. Since 0, 45, and 90 degree plies will be incorporated, the failure analysis must be performed three different times. (This is because the principal stresses will be different from the transformation of applied stresses into principal stresses.)

Using a spreadsheet (found in Appendix D) the total number of plies needed to satisfy the Azzi-Tsai-Hill and Tsai-Wu failure theories (Eqs. (9) and (10)) was found to be close to 100. This was primarily due to the relatively low value for transverse lamina strength. However, graphing the principal stresses against the number of plies for each angle is an easy way to see how many plies are needed for each orientation. Figures 5-7 show that as the number of plies increase, the principal stress decreases. Therefore, after a certain number of plies, the decrease in principal stresses tapers off significantly.

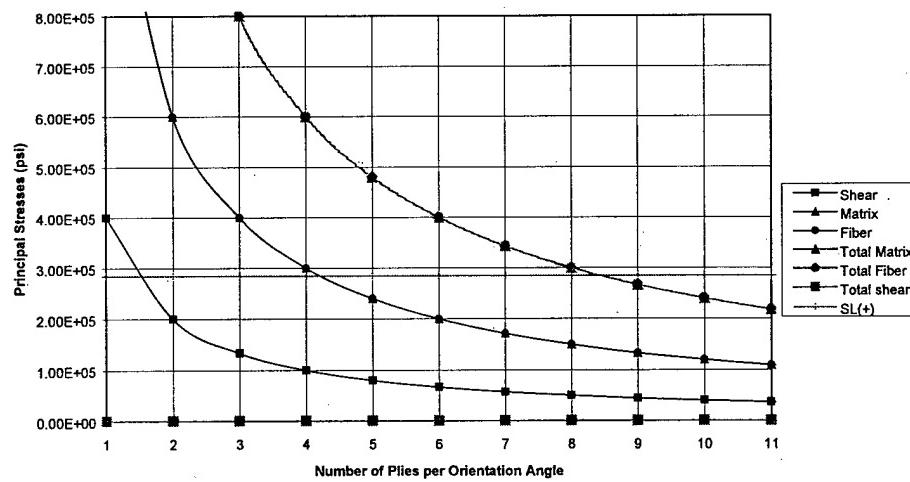


Figure 5: Principal stresses versus number of plies for an orientation angle of $\theta=45$ deg.

This means that increasing the number of plies after a certain point is not as effective. When superimposed with the Maximum Stress failure criterion, it appears to be a sensible theory to meet. Note that the total number of plies needed to meet the Maximum Stress failure theory in plies oriented at $\theta = 45^\circ$ is 9 lamina. However this only applies to the longitudinal strength, $S_{L(+)}$. Figure 6 shows that over 46 plies are needed to stay below the transverse and shear material strengths (the remaining portion of the Maximum Stress failure criterion). Approximately 100 plies are needed to meet all portions of this failure theory.) However, because the plies will be layered, it is assumed that the applied stress in the matrix direction in one ply will be carried by the fibers in a neighbor ply.

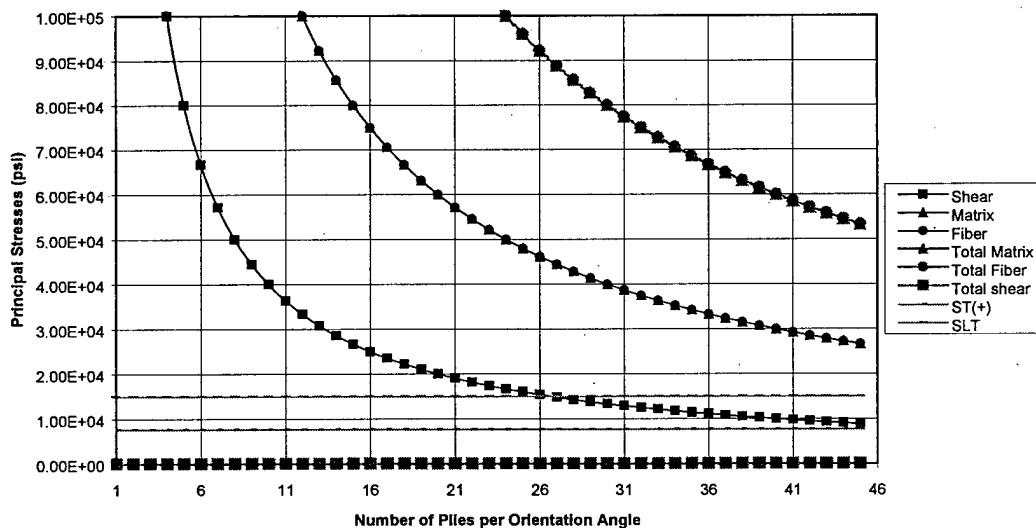


Figure 6: Principal stresses versus number of plies for an orientation angle of $\theta=45$ deg.
This is an expanded y-axis version of the previous graph.

Note that only 6 plies are needed along the axis of stress (hoop and longitudinal directions) to meet the longitudinal portion of the Maximum Stress failure theory. (Figure 6)

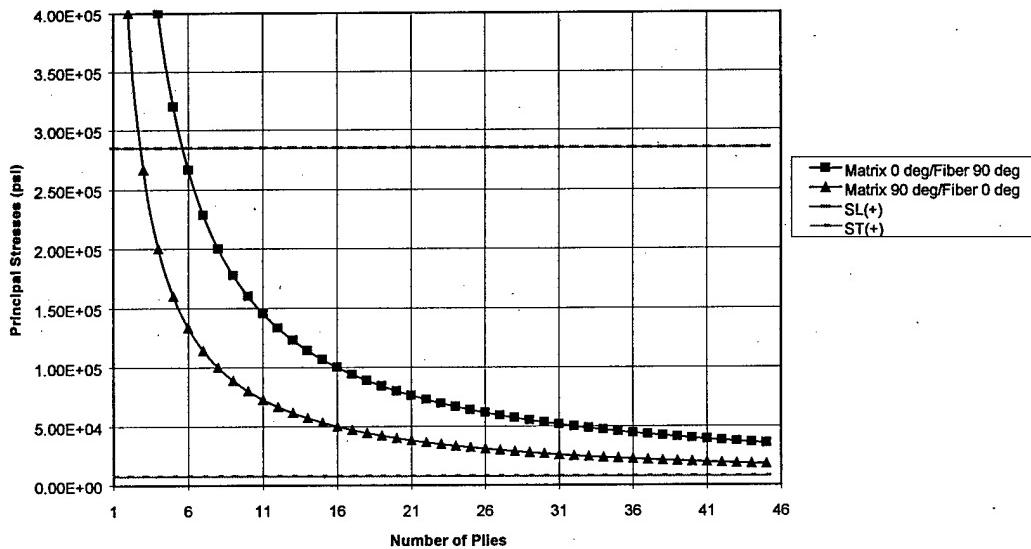


Figure 7: Principal stresses versus the number of plies for orthogonal plies.

Even though we can not meet the Azzi-Tsai-Hill or the Tsai-Wu failure criterion, we can meet the Maximum Stress failure criterion. (Therefore, for the remainder of the analysis, I will assume it is valid to select a number of layers based solely on the Maximum Stress failure theory. Spreadsheet analysis may be found in Appendix E.)

Another important aspect when analyzing how the pressure vessel will fail is geometry. As mentioned before, I have selected to design a cylindrical pressure vessel with spherical ends. Note that this is not some arbitrary design. Many composite pressure vessels are cylindrical in shape, but vary the curvature of the vessel's ends. Some vessels have elliptical ends. This could be imposed from geometry constraints or ease in manufacturing. Figure 8 shows a failed pressure vessel.

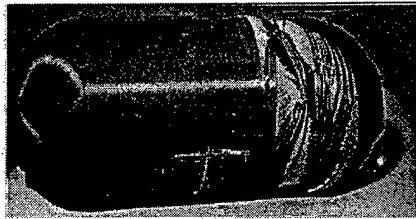


Figure 8: The character of failure of a composite pressure vessel.

From this figure we can see that the transition to elliptical ends is more extreme than a transition from spherical ends.¹⁴ Since spherical ends minimize the bending in the fibers, it is the preferred shape.

Analysis Results

Table 4 gives a summary for the composite pressure vessel design. An empirical method to size pressurant tanks for rocket systems involves the pV/W method.¹⁵ This considers the burst pressure, total volume, and mass of the vessel. From Eq. (14) the tank factor is one way to compare a tanks estimated performance.

$$\phi_{\text{tank}} = \frac{P_b V_{\text{tot}}}{g_o m_{\text{tank}}} \quad (14)$$

ϕ_{tank} = tank factor (units of length)

P_b = burst pressure

V_{tot} = vessel volume

g_o = gravitational constant

m_{tank} = vessel mass

This means that the higher the tank factor, the more pressure the vessel can hold for a given volume and mass. To put this tank factor into perspective; a typical tank factor for metallic

pressure vessels is around 2,500 meters. For a composite pressure vessel, the tank factor may be around 10,000 meters.

Table 4: Pressure Vessel Summary

Material	Gr 33/150 gsm/BT250 E-1,24 Graphite/Epoxy Unitape	
Tank geometry	Cylindrical with spherical ends Radius = 4" Length = 5' Thickness = 0.11" (just under 1/8") Volume = 50 liters	
Ply Information	Orientation (deg)	Number of Plies
	+45	5
	-45	5
	0	6
	90	6
Lay-up sequence	[0/-45/90/45/0/-45/90/45/0/-45/90] _s	
Total number of plies	22	
Approximate mass	3.74 kg	
Tank Factor, ϕ (m)	18 785	
Approximate material cost	\$ 310.00	

These results are encouraging since the current pressurant tank used by the United States Air Force Academy has a mass of 4.5 kg and a tank factor of 2800 meters. Moreover a filament wound, composite, scuba tank was one option which cost approximately \$150.00. However this tank was meant for an operating pressure of 2260 psi, had a capacity of only *9 liters*, and a mass of 4 kg. Note that this scuba tank is over-designed. The custom pressure vessel will not only weigh less, but also hold more fuel since it only has to operate at 1000 psi.

Manufacturing Analysis

Typical composite pressure vessels are made with S-glass or Kevlar 49 epoxy wrapped around a seamless metal liner (usually 6061-T6 Aluminum). This liner is used to prevent leakage through the composite or to prevent any chemical reactions. Before the vessel can be

used, it needs to be pressurized in order for the internal pressure to be carried by the composite, and not the liner.¹⁶

The composites currently used are either considered pre-impregnated (prepreg) or dry. This means the fibers either contain the resin matrix from manufacturing, or the composite must have the resin applied during lay-up. The latter refers to a wet lay-up since the user typically applies resin to the fibers before the plies are laid onto the tooling.

Pressure vessels are usually filament wound. The main benefits of filament winding are accuracy, repeatability, and low material cost.¹⁷ This method has a machine wind fibers around a metal liner or mandrel, with the resin absorbed just before lay-up. A special instrument applies the fibers in a predetermined orientation, therefore allowing the fibers to be continuous. This can be seen in figure 9.¹⁸

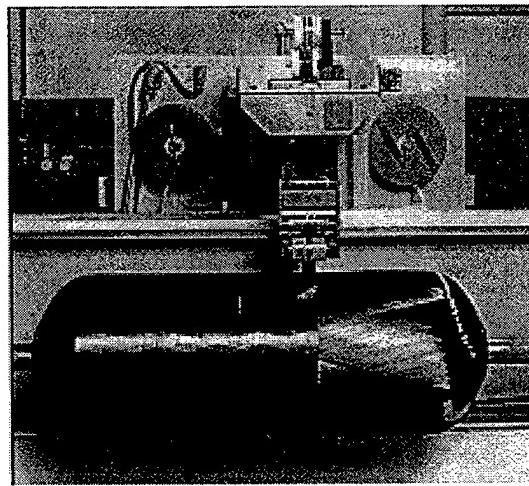


Figure 9: Large tow filament winding of pressure vessel.

One disadvantage of filament winding can be seen from Figure 9. The orientation angle is approximately 15° , which is less than the optimum 45° . I believe this was done to easily wrap the tank. If a metal liner is used, it already acts as a mandrel (a tooling device in the shape of a product used to lay up composites), and will remain in place after autoclave curing. However, if

a liner is not needed, a "wash-out" mandrel may be used. This type is considered to be like a water-soluble plaster.

For this research, I have considered to lay up a composite pressure vessel by hand. This can be referred to as fiber placement or tape laying. The main difference with tape laying is the fibers are not continuous. They are cut and laid on the mandrel in sections. Since the fibers are not continuous, there might be some loss to structural integrity. However, the advantages include reduced material scrap, while achieving an orientation angle of 45°. Since the United States Air Force Academy does not have the resources for filament winding, tape laying will be pursued. Prepreg composite is the material of choice since it contains the correct resin to fiber ratio. (The tendency with wet lay-ups is to apply too much resin which reduces curing performance.)

Many ideas came to mind when thinking about how to effectively create a mold and to tape-lay a pressure vessel. The first option for creating a mold was to use Plaster of Paris or CARE MOLD™. The latter is a dry powder from Composite Horizons, Inc. This material is specifically mixed with water to form a mandrel. When cured, this material can withstand typical autoclave conditions. The unique property of CARE MOLD™ is that it can be washed-out with pressurized water. This feature does not hold true for Plaster of Paris, which acts like cement when cured. The bottom line between the two options is cost. Plaster of Paris usually runs about \$7.00 for a 25-pound bag. However CARE MOLD™ runs about \$250.00 for a 50 pound bag. Furthermore, CARE MOLD™ contains crystalline silica, which is a carcinogenic. More information on this product may be found in Appendix F.

Another option for designing a mold was to either blow glass or plastic into a properly shaped vessel. These mandrels would ideally act as a liner, however the glass liner might not

withstand launch vibrations, while the plastic liner might have fuel compatibility limitations. The one advantage for the glass mandrel is it may be easily broken out of the vessel, which makes it quite an attractive option for pressure vessels not needed a liner. Allen Scientific Glass gave price estimates for some prototype and actual pressure vessel sizes. Note that this company can make a 7" diameter glass mandrel for a reasonable price. However, the price gets very expensive for an 8" diameter glass mandrel because of material availability. Table 5 summarizes the cost and benefits from each material.

Table 5: Mold Product Summary

Product	Cost (\$/lb)	Advantages	Disadvantages
Plaster of Paris	0.2788	Inexpensive, Non-Toxic	Not dissolvable, Difficult to break
CARE MOLD™	5	Water soluble	Expensive, Carcinogenic
Glass	Cost/product (\$)	Inexpensive	Never tried
3"x10"	25.00		
7"x3'	135.00		
7"x5.5'	200.00		

The last option was to consider a wax mold. This wax would have to withstand the curing temperature and pressure of the autoclave. If this happens then the wax can be melted or dissolved from the pressure vessel.

When deciding how to lay-up the pressure vessel two ideas came to mind. These included tape-laying a whole pressure vessel, or half of one. If I was to lay-up a whole pressure vessel, the best dissolvable mandrel is appropriate. However if I was to lay-up half of a pressure vessel (construct vessel in sections), then I could use the best suited mandrel (which might be Plaster of Paris because it is the least expensive). Table 6 give a trade summary for a sectioned and whole pressure vessel.

Table 6: Construction Summary Comparison

Sectioned	Whole
Simple tooling	Constrained to "wash out" mandrel
Consideration for section bonds	Consideration for careful tape laying
Reusable tooling	Difficulty in removing tooling? Non-reusable tooling Integrity of whole vessel

Procedure

My goal for tape laying this pressure vessel is to determine the effects of thermal expansion differences on spherical and sharply curved surfaces. Therefore a six-ply prototype was manufactured. I felt this was sufficient layers to conclude any effects of thermal expansion around any curved surfaces. Furthermore, I wanted to conserve as much material as possible and still conclude if it is worth pursuing the manufacturing process of choice.

Regular soda bottles were used as a makeshift female mold for both the plaster and the CARE MOLD™. From Figure 10, one bottle was cut in half so that the mold may be filled easily with plaster. The Plaster of Paris was easily removed and exhibited a smooth finish. This male mold was easily sanded or carefully sawed to produce the proper end geometry.



Figure 10: Female mold used for Plaster of Paris

A test was done using an autoclave to see the effects of temperature and pressure ramping on this male mold. Using a previous program, which cycled at 80 psi and 320 °F, the plaster mold cracked at the neck. (Refer to the end of Appendix C for the cure cycle.) This was because of

the non-uniformity it mating surfaces between the pressure plate and the plaster mold. This pressure plate acted as a backing, which the vacuum bag sealed to. Figure 11 shows how the plaster mold was vacuum bagged. However, there were small hairline cracks throughout the mold. (The neck cracking in the first place might have caused this, so I decided to analyze this plaster mold even further.)

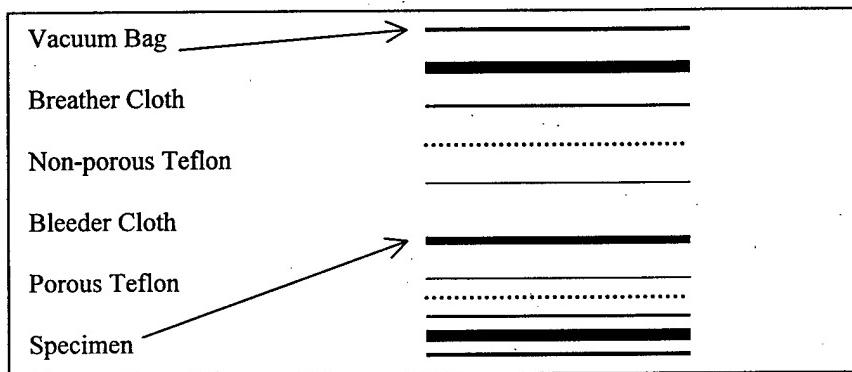


Figure 11: Sequence for preparing composite a specimen for autoclave curing. Note that the sequence is symmetric about the specimen (for curing cylinders only.)

The CARE MOLD™ male mold exhibited different properties than expected. This material claims to "have excellent thermal and casting properties...to eliminate the need for corrections in the mold for shrinkage or thermal expansion. CARE MOLD 800 is washed out with pressured water and agitation. Drilling or casting removable rods into the mandrel will facilitate in the washing out of the mold." This substance was found to be difficult to break apart and dissolve in water, which is contradictory to what the Composite Horizon Inc. claims about CARE MOLD™. Even though it *does* dissolve in water, it will not dissolve the mold without work. This causes some concern if the neck diameter is much smaller than the tank diameter. In addition, CARE MOLD™ acted like paste, which made it difficult to insert into the mold with a small opening. Lastly, CARE MOLD™ seemed to exhibit a low yield, which was gathered from the first test sample.

Once the two molds were made, I decided to first examine a lay-up on the plaster mold. If the lay-up procedure was a success, then I could move on to joining the pieces to form a whole pressure vessel. If I succeeded at this, then finally I could test the vessel (after a proper pressure fitting was incorporated) to establish some performance characteristics. If the plaster mold outperformed CARE MOLD™, then money would be saved in the tooling.

In order to establish ply geometry, I first worked with sheets of paper. Using various shapes, I was able to determine that it might be possible to separately lay up the vessel's spherical and cylindrical sections without crimping the fibers. These sections would overlap per ply and create a joint between the spherical and cylindrical sections (Figure 12).

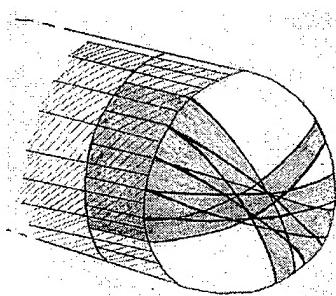


Figure 12: Possible ply geometry. The middle band refers to the joining of cylindrical and spherical tape pieces.

Once the ply geometry is attained, they need to be cut from the roll of graphite/epoxy unitape. This is not trivial, and Appendix G shows the actual measurements used to cut the plies from the roll. Since the composite material has resin mixed in already, it must be stored around 32°F, otherwise the composite material will start to cure, reducing the shelf life.

Care must be taken when laying up the pressure vessel. As stated earlier in Table 4, the lay-up sequence is [0/-45/90/45/0/-45/90/45/0/-45/90]. For any composite structure, the lay-up needs to be symmetric about the centerline for the laminate. This is because; "Thermal loads

appear due to restrictions imposed by various layers against their free thermal expansion."¹⁹ This results in mid-plane stresses and curvatures. Plies tend to contract more in the matrix direction (transversely across the fibers), and residual thermal stresses may develop due to a variation in adjacent ply orientation.²⁰ This holds true for the specimen at the end of the cure cycle, while it is cooling down. However, if the laminate is symmetric about its centerline, a mirror image would be created about the mid-plane, making the effects of residual thermal stresses negligible on laminate shape. Furthermore, to reduce the free edge effects of laminates (i.e. fraying) a proper lamina stacking sequence is needed. Since the laminate (pressure vessel) will contain 0°, 90°, and ±45° layers, adjacent +45° and -45° should be avoided.²¹

A technician at the mechanical laboratory at the United States Air Force Academy, Jeff Logston, was able to reprogram the autoclave with the correct cure cycle program. To make sure it was running correctly, thermocouples were used to verify the actual temperature inside the autoclave with the digital read-out. Detail on the cure cycle for the graphite/epoxy may be found at the end of Appendix C.

Manufacturing Results

After the cure cycle was running properly, and the plies were cut, three prototypes were tested. The first prototype consisted of a small wrapping of graphite/epoxy around a small test sample of CARE MOLD™. This served three purposes; to determine how CARE MOLD™ and the composite material handled under the cycle, and to determine how the two interacted with each other.

When removed from the autoclave, the first observation was the absence of resin absorbed by the bleeder cloth. Even though CARE MOLD™ handled the pressure and

temperature of the cycle, the composite did not cure as expected. Two questions arose from these results. Is the graphite/epoxy defective (i.e. is there not enough resin in the material), or did the CARE MOLD™ soak up the resin?

Therefore, the second prototype was designed to answer these questions. A 32 ply, symmetric coupon was made and tested at the cure cycle. (A high number of plies was used to make certain resin would flow throughout all lamina.) The coupon came out of the cycle correctly cured. The bleeder cloth contained excess resin, which was expected, and the laminate was extremely stiff. Furthermore, no fibers could be removed because the matrix properly cured; all of which leads me to believe the material is not defective, and that CARE MOLD™ absorbed the resin. This can be seen from the discoloration in the mold where the composite was wrapped. This means that for future use, the CARE MOLD™ must be sealed properly to prevent the resin penetrating under a vacuum.

The final prototype's goal has many facets. First I wanted to see how a better-designed plaster mold would handle the pressure and temperature of the cure cycle. I also wanted to examine the effects of resin absorption. (For this, I wrapped part of the mold in non-porous Teflon and sprayed the remainder with releasing agent.) Next, I wanted to see the feasibility of tape-laying the mold with the previously mentioned ply geometry. (Appendix G) Finally, I wanted to make a conclusion about the effects of random ply overlap (for a given orientation) on final cure shape. (This is important because of the residual stresses from thermal expansion variations as mentioned earlier.)

Six plies were layed up around the non-porous Teflon section of the plaster mold where the sequence was [90/-45/0]_s. Because of difficulty in tape laying, only three plies were wrapped around the bare mold (sprayed with releasing agent), which included the mold's neck. However,

this was sufficient for testing purposes. To prevent the neck from cracking, I applied vacuum bagging tape to create a cushion between the mold and the pressure plate.

An abundance of information was obtained from this prototype. First, the plaster mold withstood the cure cycle's pressure and temperature. There were no hairline cracks to be found on the mold, which means Plaster of Paris exhibits a low coefficient of thermal expansion-attractive for this cure cycle. Second, the section of the mold that was sprayed with releasing agent absorbed resin. Therefore, a sealing agent is needed to prevent resin seeping into the plaster mold. Teflon worked well, but was difficult to wrap around a spherical surface. Note that the plaster mold wrapped with Teflon was removed with no damage. This means that only one mold is needed for multiple lay-ups, since the mold was not destroyed in the separation process. It would be easier to use some type of spray-on sealing agent for spherical surfaces. The outer surface of the final prototype displayed some wrinkling. This is most likely due to the plies not seating properly against the mold. This is not difficult to work around as long as more care is taken in the lay-up process. Furthermore, variations in thermal expansion posed no threat even with random overlapping. The prototype was not warped which meant that this lay-up procedure has a high potential for success. (This can only be known after a completed prototype is tested under pressure.)

The three plies layed up around the neck tell a different story. It was troublesome to get the plies to lay smoothly around such tight curves. In addition, it was difficult to get the vacuum applied evenly around the neck area. This resulted in improper curing. Therefore, a different approach is needed to successfully wrap around tight curves. Finally, these results conclude it is still feasible to use a plaster mold and create a pressure vessel from sections.

Conclusion

There are still many avenues to pursue with this research. Some of the following ideas might help in determining a solution to a lightweight, pressure vessel for a hybrid sounding rocket. First see how a glass mold prototype (or even flour and water prototype) would effect the current lay-up process. Even a collapsible mandrel can be investigated for wrapping either a whole or partial vessel. Of course this mandrel would not be the kind to wash out, making it reusable. Not only would this be a one-time tooling cost, but also would improve the consistency and repeatability for making multiple vessels.

I also suggest deciding on a way to incorporate a pressure fitting with a goal of reducing the pressure vessel's curves. This will help with lay-up and curing. The plaster mold could even be shaped according to Figure 13.

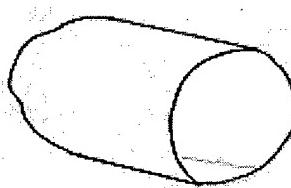


Figure 13: Alternate female mold
for Plaster of Paris

Furthermore, the cure cycle could be modified in such a way to reduce any residual stresses by reducing the cure temperature and increasing the cure time. These ideas might help with the lay-up process and bonding between sections. Modification could even be made to the ply geometry to minimize the number of discontinuous fibers.

Another idea is to create the pressure vessel in deliberate sections. For example, stamp the spherical sections using a pressure mold or die. This may reduce the crimping of unidirectional hand lay-ups on spherical surfaces. The cylindrical portion would be constructed

like a composite tube; however, both spherical and cylindrical sections would have flanges.

These flanges would aid in joining the sections by incorporating epoxy, bolts, or o-rings, or any combination of the three.

With these options for making a tape-layed pressure vessel, the goal is to test each option and determine which vessel achieved the best performance. This performance is most likely based on mold cost, lay-up process, surface finish, and operating pressure. In order to test a prototype of different dimensions, one question needed to be answered is: What is an equivalent prototype pressure, and how is this related to the operating pressure for the pressure vessel of actual size? Using dimensional analysis, Eq. (15) yields:

$$p_2 = p_1 \left(\frac{n_2 \sigma_{h2} t_2 r_1}{n_1 \sigma_{h1} t_1 r_2} \right) \quad (15)$$

p_1 = pressure in actual vessel

p_2 = pressure in prototype vessel

n_1 = number of plies in the actual vessel

n_2 = number of plies in the prototype vessel

σ_{h1} = hoop stress in actual vessel

σ_{h2} = hoop stress in prototype vessel

$t_{1,2}$ = actual and prototype vessels total thickness respectively

$r_{1,2}$ = actual and prototype vessel radius respectively

If the hoop stress, total thickness, and number of plies are assumed to be constant, then equation 15 reduces to just the ratio of vessel radii. (Appendix H)

Two destructive testing ideas came to mind for evaluation. Once fitted with a pressure fitting, the prototype vessel can be pressurized under water. This will establish performance characteristics such as porosity and operating pressure. Another way to test only operating pressure is to apply strain gauges and determine the actual amount of longitudinal and hoop stress being produced. However, destructive testing is not always beneficial because no one

could use the pressure vessel if it did hold 2000 psi. Therefore electromagnetic testing is one viable option for non-destructive testing. (Ultrasonic testing would not work for pressure vessels since sound waves could not be effectively reflected off curved surfaces.) With these methods, conclusions can be made about the prototype's performance.

Since the actual pressure vessel is about 5.5 feet long, it will not fit in the autoclave at the United States Air Force Academy. One method to work around this obstacle is to vacuum bag the specimen with a portable vacuum pump, and then create a container from insulative materials to ramp up the temperature. Using principles from heat transfer, thermocouples, and fans, a makeshift autoclave can be designed for this actual pressure vessel. [This is what members of a FSAE design team did to make a full-size composite tub for a formula type race car at Syracuse University in 1997.]

However, if none of the prototypes successfully hold the operating pressure, all is not lost. A manufacturing method has been reached which could easily be applied to tubes. Tubes can be manufactured if the lay-up does not include spherical ends. These tubes can be used for a multitude of options. Either structural members, or outer body sections are two applications for the hybrid sounding rocket.

Acknowledgements

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References

- ¹ Gibson, R. F., Principles of Composite Material Mechanics, McGraw-Hill Inc., St. Louis, 1994.
- ² Ashby, M. F., and Jones, D. R. H., "The Price and Availability of Materials," Engineering Materials 1 An Introduction to Their Properties and Applications, Pergamon Press Inc., New York, 1993, pp. 13-21.
- ³ Op. Site n. 2
- ⁴ Hazen, J. R., "Expanding Applications Reinforce the Value of Composites," High Performance Composites Sourcebook 1998, Vol. 5, No. 6, November 1997, pp. 10-25.
- ⁵ Beer, F. P., and Johnston, E. R., "Transformations of Stress and Strain," and "Stresses in Thin-Walled Pressure Vessels," Mechanics of Materials, 2nd ed., McGraw-Hill Inc., San Francisco, 1992, pp. 339-347, 377-380.
- ⁶ Op. Site n. 5
- ⁷ Op. Site n. 1
- ⁸ Mallick, P. K., "Stress-Strain Relationships for a Thin Lamina," Fiber-Reinforced Composites Materials, Manufacturing, and Design, 2nd ed., Marcel Dekker Inc., New York, 1993, pp. 133-140.
- ⁹ Christensen, R. M., Mechanics of Composite Materials, John Wiley & Sons Inc., New York, 1979.
- ¹⁰ Op. Site n. 9
- ¹¹ Op. Site n. 8
- ¹² Op. Site n. 1
- ¹³ Cook, R. D., and Young, W. C., "Failure and Theories of Failure," Advanced Mechanics of Materials, Macmillan Publishing Co., New York, 1985, pp. 21-26.
- ¹⁴ Vasiliev, V. V., and Jones, R. M., Mechanics of Composite Structures, Taylor & Francis Publishing, Washington D.C., 1993.
- ¹⁵ Humble, R. W., Henry, G. N., and Larson, W. J., "Estimating Tank Mass With the pV/W Method," Space Propulsion Analysis and Design, McGraw-Hill Inc., New York, 1995, pp. 272-273.
- ¹⁶ Op. Site n. 8

¹⁷ Op. Site n. 4

¹⁸ Op. Site n. 4

¹⁹ Op. Site n. 8

²⁰ Op. Site n. 8

²¹ Op. Site n. 8

Appendix

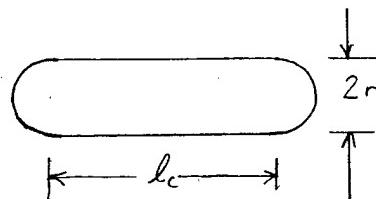
- A: Calculations for shape and size of actual tank
- B: Review of mechanics on pressure vessels
- C: Information on Gr33 150 gsm/BT250E-1, 24 graphite/epoxy unitape
- D: Actual optimization of ply orientation and spreadsheets
- E: Spreadsheet data for Figures 5-7: Principal stresses versus number of plies
- F: Material information on composites and CARE MOLD™
- G: Composite roll measurements
- H: Dimensional analysis on testing pressure

- Tank geometry based on Nitrous Oxide, with a cylindrical shape with spherical ends

- Given: $m = 20 \text{ kg}$

$$\rho = 400 \text{ kg/m}^3$$

$$r = 4 \text{ in} = 0.1016 \text{ m}$$



- Find total length

$$V = \frac{m}{\rho} = \frac{4}{3}\pi r^3 + \pi r^2 l_c$$

$\frac{4}{3}\pi r^3$
Spherical Section
 $\pi r^2 l_c$
Cylindrical Section

$$l_c \pi r^2 = \frac{m}{\rho} - \frac{4}{3}\pi r^3$$

$$l_c = \frac{1}{\pi r^2} \left(\frac{m}{\rho} - \frac{4}{3}\pi r^3 \right)$$

- For one tank

$$l_c = \frac{1}{\pi (0.1016)^2} \left(\frac{20}{400} - \frac{4}{3}\pi (0.1016)^3 \right) \text{ m}$$

$$l_c = 1.4063 \text{ m} \Rightarrow 4 \text{ ft } 7\frac{1}{2} \text{ in}$$

- For two tanks of equal size and shape

$$l_c' = \frac{1}{\pi (0.1016)^2} \left(\frac{10}{400} - \frac{4}{3}\pi (0.1016)^3 \right) \text{ m}$$

$$l_c' = 0.63544 \text{ m} \Rightarrow 2 \text{ ft } 1 \text{ in}$$

One Tank

$$l_T = l_c + 2r \\ = 1.6095 \text{ m}$$

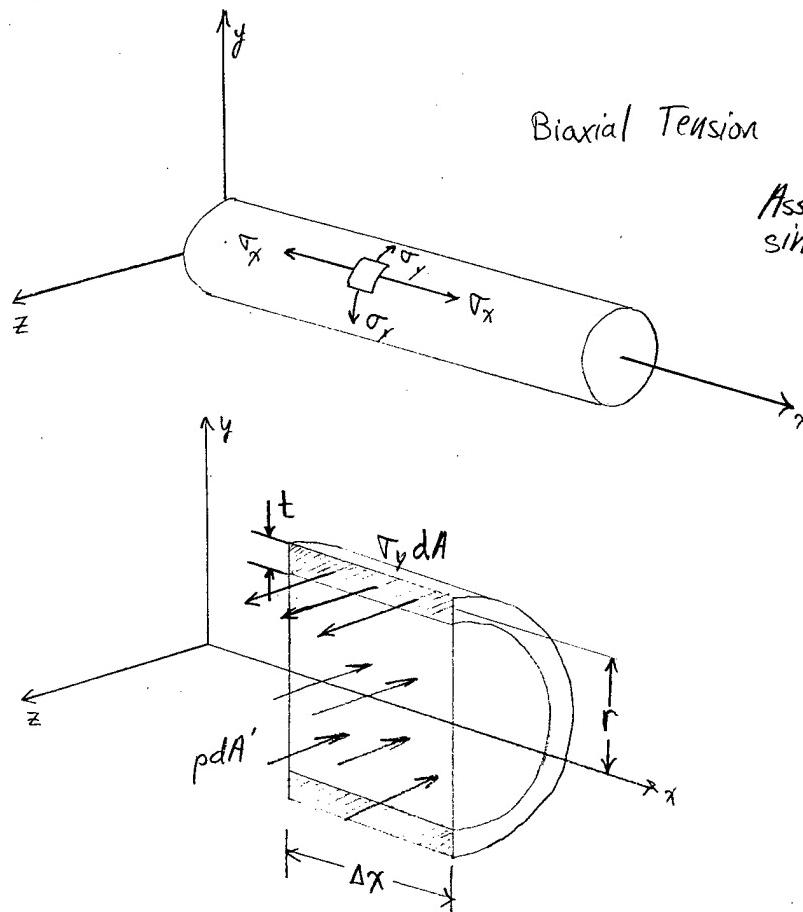
$$l_T \approx 5 \text{ ft}$$

Two Tanks

$$l_T' = l_c' + 2r \\ = 0.83864 \text{ m}$$

$$l_T' \approx 2 \text{ ft } 5 \text{ in}$$

590 SLEEVES, FULLER 5 SQUARE
590 SLEEVES, FULLER 5 ASSY
42-381 502 SLEEVES, FULLER 5 ASSY
42-382 502 SLEEVES, FULLER 5 ASSY
42-383 500 RECYCLED WHITE 5 SQUARE
42-384 200 RECYCLED WHITE 5 SQUARE
42-385 500 RECYCLED WHITE 5 SQUARE
42-386 500 RECYCLED WHITE 5 ASSY
42-387 500 RECYCLED WHITE 5 ASSY
42-388 500 RECYCLED WHITE 5 ASSY
42-389 500 RECYCLED WHITE 5 ASSY
Made in U.S.A.



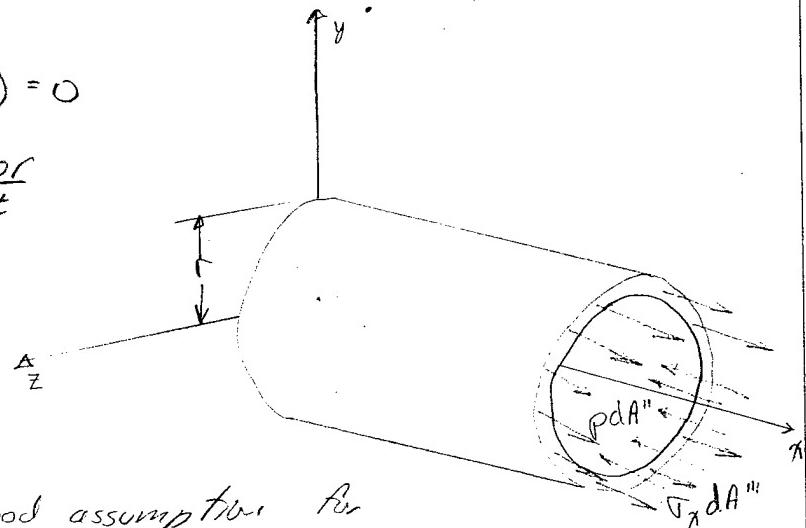
$$dA = t \Delta x$$

$dA' = 2r \Delta x$ → Note since $r \gg t$, $\frac{t}{2}$ is negligible

$$\sum F_z = 0$$

$$T_y(2t \Delta x) - p(2r \Delta x) = 0$$

$$T_y = \frac{p(2r \Delta x)}{2t \Delta x} = \frac{pr}{t}$$



$$dA'' = \pi r^2$$

$dA''' = 2\pi r t$ → Good assumption for small t .

$$\sum F_x = 0$$

$$T_x dA''' - p dA'' = 0$$

$$\sigma_x (2\pi r t) - p(\pi r^2) = 0$$

$$\sigma_x = \frac{p(\pi r^2)}{2\pi r t} = \frac{pr}{2t}$$

More accurately:

$$\sigma_x = \frac{pr}{2t} \frac{1}{\left(1 + \frac{t}{2r}\right)}$$

Out of Plane Stress

$$\tau_{xy} = \frac{pr}{2t} \left(1 + \frac{t}{r}\right)$$

13-782 500 SHEETS, FILLER, 5 SQUARE
42-381 50 SHEETS EYE-ASA, 5 SQUARE
42-382 100 SHEETS EYE-ASA, 5 SQUARE
42-389 200 SHEETS EYE-ASA, 5 SQUARE
42-392 100 RECYCLED WHITE, 5 SQUARE
42-398 200 RECYCLED WHITE, 5 SQUARE
Made in U.S.A.





Bryte Technologies, Inc.
2025 O' Toole Ave.
San Jose, CA 95131
(408) 434-9809
FAX: (408) 434-9811

Certificate of Material Conformance

Customer

Certification Number: 10328
Shop Order Number: 24362
Date: 12/8/97

Company: Advanced Composite Technologies

Buyer: Teresa Marian

PO #: 97-1377

Resin Spec: N/A

Fabric Spec: N/A

Product

Product: Gr 33 150 gsm/BT250E-1, 24 *25°/lb 21.5 sq ft*

Description: Graphite Epoxy Unitape, 150 gsm FAW, 24" width, 250°F Cure.

Fiber Type: Gr 33 Graphite

Resin Type: Epoxy, BT250E-1

Manufacturing Information

Run/Mix Lot Number: 120197-2T2

Date of Manufacture 12/04/97
(DOM):

Measured: Resin Content **Values:** Roll #1: 39.4%

Specified Resin Content : 40±3%

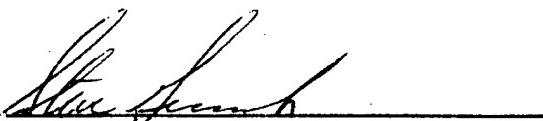
Shelf Life
(from date of
shipment): 30 days @ 77° F _____
 12 months @ - 0° F _____

Quantity
in Spec: 19.3 lbs

COMMENTS:

Certified by

Quality Assurance

A handwritten signature in black ink, appearing to read "Teresa Marian". It is placed over a horizontal line that extends from the end of the "Certified by" text.

BT250E-1 Resin System

PRODUCT TYPE

250°F Cure
Epoxy Resin System

SERVICE TEMPERATURE

200°F (Continuous)

TYPICAL APPLICATIONS

- Secondary Aircraft Structures
- Racing Vehicles
- Radomes with Spectra®, Glass, Quartz & Kevlar®
- Reflectors
- Sporting Goods
- Knee Braces and Other
- Related Medical Items
- General Purpose Composites

PRODUCT DESCRIPTION

The BT250E-1 resin system is unique in that it displays good toughness and strength in a standard epoxy matrix. Its chemistry along with Bryte's Proprietary OHMS impregnation process provide a system that displays an outstanding surface finish with lower case vacuum bag/oven cure only. The resin system, which is self adhesive to honeycomb and foam core, is Mil-R-9300 qualified and makes a great choice for many applications in the low to medium service temperature range.

MECHANICAL PROPERTIES
7781 "E" Fiberglass Reinforcement

Tensile Strength	62.0 ksi
Modulus	2.7 Msi
Compressive Strength	69.0 ksi
Modulus	3.1 Msi
Flexural Strength	70.0 ksi
Modulus	3.0 Msi
Short Beam Shear Strength	7.7 ksi

5.1/5 Graphite Reinforcement (3.4 Msi)

Tensile Strength	97.0 ksi
Modulus	9.0 Msi
Compressive Strength	85.0 ksi
Modulus	9.1 Msi
Flexural Strength	133.0 ksi
Modulus	9.0 Msi
Short Beam Shear Strength	8.5 ksi

dielectric
constant
7781

Graphite (3.4 Msi) Unidirectional Reinforcement

Tensile Strength	285 ksi
Modulus	19.0 Msi
Compressive Strength	245 ksi
Modulus	18.5 Msi
Flexural Strength	260 ksi
Modulus	19.2 Msi
Short Beam Shear Strength	15.0 ksi

All data given is based on representative samples of the materials in question. Since the method and circumstances under which these materials are processed and tested are key to their performance, and Bryte Technologies Inc. has no knowledge of how its customers will use the material, the corporation cannot guarantee these properties.



Product Defect Log

Bryte Technologies, Inc.
2025 O' Toole Ave.
San Jose, CA 95131
(408) 434-9809
FAX: (408) 434-9811

Company: Advanced Composite Technologies

PO #: 97-1377

Product: Gr 33 150 gsm/BT250E-1, 24

Description: Graphite Epoxy Unitape, 150 gsm FAW, 24" width, 250°F Cure.

Order #: 24362

Roll #: 11

Lot #: 120197-212

Date Of Manufacture: 12/4/97

Quantity in Spec: 19.3 lbs

Defect Type	Defect Size	Location *	Material Allowed
1 BT	1" x 1"	58	1
2			
3			
4			
5			
6			
7			
8			
9			
10			
11			
12			
13			
14			
15			

TOTAL ALLOWED: _____

* All defect locations are noted from the "core end" outward and marked for customer convenience.

Legend:	RR - Resin Rich	S - Split/Gap	D - Debris	BT - Broken Tow
	RS - Resin Starved	W - Wrinkles	F - Fabric Defect	FB - Fuzz Ball
	SM - Seam/Splice	T - Tear	P - Pinch Lift	PK - Pucker
	FC - Fiber Crossover	MT - Missing Tow	FA - Fiber Alignment	



Bryte Technologies, Inc.
2025 O' Toole Ave.
San Jose, CA 95131
(408) 434-9809
FAX: (408) 434-9811

MATERIAL SAFETY DATA SHEET **(PRELIMINARY INFORMATION)**

REVISION DATE: 04/14/97

SECTION I: MANUFACTURER AND MATERIAL

MANUFACTURER: BRYTE TECHNOLOGIES INC
2025 O'TOOLE AVE.
SAN JOSE, CA 95131

EMERGENCY TELEPHONE: 408.434.9809

MATERIAL TRADE NAME: **BT250E-1 Prepreg, 250°F Cure**

CHEMICAL FAMILY: Epoxy Resin Impregnated Material; Fiberglass, Graphite, Quartz, Aramid, Ceramic, or Oriented Polyethylene Fibers

FORMULA: Proprietary

PRODUCT INGREDIENTS: Bisphenol A Epoxy Resin, Proprietary Curing Agent, Fiber Reinforcement

SECTION II: HAZARDOUS INGREDIENTS

CAS#	
14808-60-7	Fiberglass*, Ceramic*, or Quartz* Fiber
7782-42-5	Graphite* Fiber
25068-38-6	Bisphenol A/Epichlorohydrin Epoxy Resin

*Fiberglass, Quartz, Graphite, Aramid, Polyethylene, or Ceramic fibers (>50%), are a "nuisance particulate" not otherwise regulated, for dust and potential exposure during machining of cured product.

SUSPECTED CANCER CAUSING AGENTS: None

SECTION III: PHYSICAL DATA

BOILING POINT (F):	N/A	SPECIFIC GRAVITY:	>1
VAPOR PRESSURE (mm Hg):	N/A	PERCENT VOLATILE:	<2
VAPOR DENSITY (air=1):	N/A	EVAPORATION RATE:	N/E
SOLUBILITY in WATER:	Negligible	MELTING POINT:	N/A
ODOR:	None	D.O.T. HAZARD CLASS:	Not Regulated
APPEARANCE:	Fabric impregnated with clear milky resin		

SECTION IV: FIRE AND EXPLOSION HAZARD DATA

Flash Point:	N/A	FLAMMABLE LIMITS:	N/A
LEL:	---	UEL:	---
EXTINGUISHING MEDIA:	CO ₂ , Dry Chemical, Foam or Water.		
SPECIAL FIRE FIGHTING PROCEDURES:	Self Contained Breathing Apparatus and Protective Clothing should be worn in all fires involving Chemicals.		
UNUSUAL FIRE AND EXPLOSION HAZARDS:	Exothermic Polymerization can occur with rapid or excessive heat. When heated to decomposition, toxic fumes are emitted. Incineration can generate airborne graphite fibers which may cause electrical malfunctions, when graphite is the fiber reinforcement.		

SECTION V: HEALTH HAZARD DATA

THRESHOLD LIMIT VALUE:	None established for this product. No carcinogenicity: NTP, IARC, OSHA.
EFFECTS OF OVEREXPOSURE:	Prolonged or repeated contact may cause skin irritations. Vapors released during product cure may cause irritation to the eyes and/or respiratory system. Dust from machining operations of cured product may cause irritation to the eyes and/or respiratory system.
EMERGENCY & FIRST AID PROCEDURES:	In the event of skin problems, thoroughly wash the affected areas with soap and water. If eyes are affected, flush with water for at least 15 minutes and obtain medical assistance if the irritation persists. Provide oxygen and obtain medical assistance if there are any adverse effects from inhalation of curing vapors.

SECTION VI: REACTIVITY DATA

STABILITY: Stable For curing process only.

INCOMPATIBILITY (Materials to Avoid): Strong Acids, Bases, Oxidizing Agents.

HAZARDOUS DECOMPOSITION PRODUCTS: Hazardous decomposition by-products may include CO₂, CO, aldehydes, nitrogen oxides, trace HCl.

HAZARDOUS POLYMERIZATION: May occur if a large mass of material is subjected to rapid or excessive heat. Condition will not occur under normal processing parameters

CONDITIONS TO AVOID: Rapid or Excessive Heat.

SECTION VII: SPILL OR LEAK PROCEDURES

MATERIAL IS RELEASED N/A
OR SPILLED:

WASTE DISPOSAL METHOD: Consult certified waste disposal contractor for disposal in accordance with all Federal, State, and local regulations.

SECTION VIII: SPECIAL PROTECTION INFORMATION

RESPIRATORY PROTECTION: NIOSH approved organic vapor respirator recommended when heating > 120°F. NIOSH approved dust mask worn when machining cured product.

VENTILATION: Local Exhaust, to control vapors or dust generated. Mechanical (General), Local preferred.

PROTECTIVE GLOVES: Impervious Gloves (Latex).

EYE PROTECTION: Safety Glasses.

OTHER PROTECTIVE EQUIPMENT: Eyewash fountains. Barrier creams.

SECTION IX: SPECIAL PRECAUTIONS

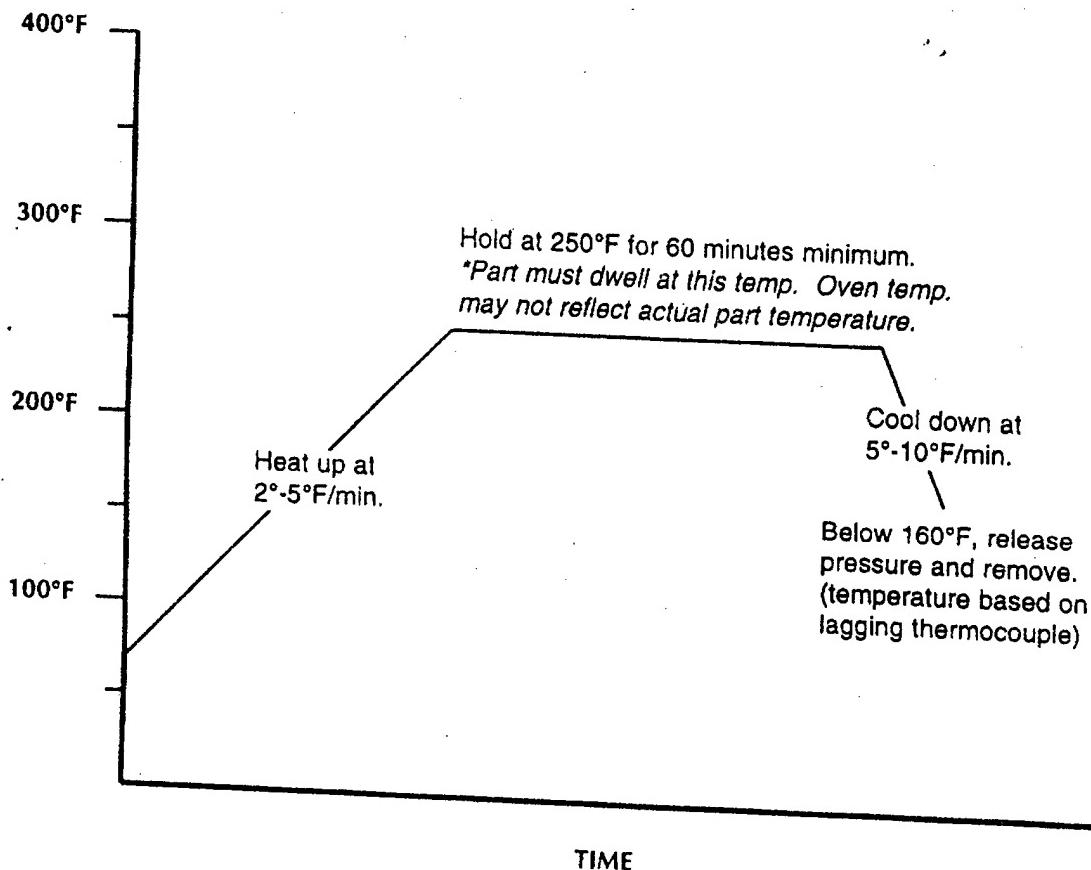
HANDLING AND STORING: Maintain sealed against contamination from dirt and moisture. Store below 10°F. Airborne graphite fibers and/or dust can create a severe electrical short hazard.

BT250E-1 Epoxy Prepreg System



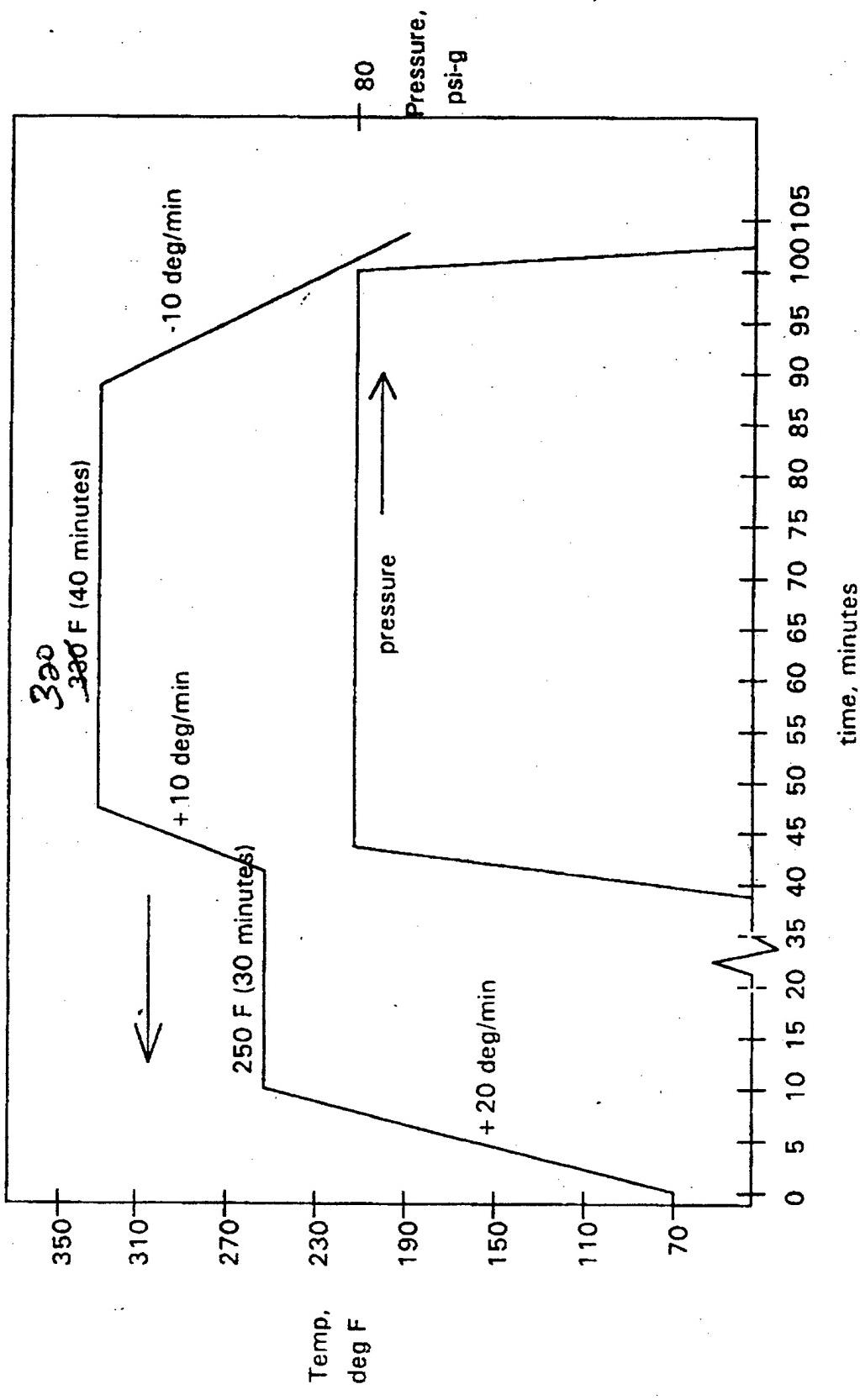
Rev. 594

Cure Cycle



- Apply 25 inches Hg vacuum minimum.
- Apply 30 - 100 psig pressure to autoclave (optional).

Program 19: Cure Cycle for 3M SP-1003 E glass/epoxy prepeg



- Let σ_h = hoop stress = σ_y
 σ_l = longitudinal stress = σ_x
 σ_m = stress in matrix direction (transverse stress) = σ_z
 σ_f = stress in fiber direction = σ_1
 τ_{hl} = shear stress in h, l frame = τ_{xy}
 τ_{fm} = shear stress in f, m frame = τ_{12}

$$\sigma_l = \sigma_f \cos^2 \theta + \sigma_m \sin^2 \theta - 2 \tau_{fm} \sin \theta \cos \theta$$

$$\sigma_h = \sigma_f \sin^2 \theta + \sigma_m \cos^2 \theta + 2 \tau_{fm} \sin \theta \cos \theta$$

or

$$\sigma_1 = \sigma_x \cos^2 \theta + \sigma_y \sin^2 \theta + 2 \tau_{xy} \sin \theta \cos \theta$$

$$\sigma_2 = \sigma_x \sin^2 \theta + \sigma_y \cos^2 \theta - 2 \tau_{xy} \sin \theta \cos \theta$$

$$\tau_{12} = -\sigma_x \sin \theta \cos \theta + \sigma_y \sin \theta \cos \theta + \tau_{xy} (\cos^2 \theta - \sin^2 \theta)$$

$\tau_{xy} = 0$ Because we assume biaxial tension

$$\begin{aligned} \sigma_1 &= \sigma_x \cos^2 \theta + \sigma_y \sin^2 \theta & \sigma_x &= \sigma_1 \cos^2 \theta + \sigma_2 \sin^2 \theta \\ \sigma_2 &= \sigma_x \sin^2 \theta + \sigma_y \cos^2 \theta & \sigma_y &= \sigma_1 \sin^2 \theta + \sigma_2 \cos^2 \theta \end{aligned}$$

$$\text{Since } \sigma_x = \frac{\sigma_y}{2}$$

$$\frac{\sigma_y}{\sigma_x} = \frac{\sigma_1 \sin^2 \theta + \sigma_2 \cos^2 \theta}{\sigma_1 \cos^2 \theta + \sigma_2 \sin^2 \theta}$$

$$2 = \frac{\sigma_1 \sin^2 \theta + \sigma_2 \cos^2 \theta}{\sigma_1 \cos^2 \theta + \sigma_2 \sin^2 \theta}$$

If $\sigma_1 \gg \sigma_2$

$$2 = \frac{\sigma_1 \sin^2 \theta}{\sigma_1 \cos^2 \theta} = \frac{\sin^2 \theta}{\cos^2 \theta}$$

$$\therefore \theta \approx 54.74 \text{ deg}$$

13-782
500 SHEETS, FELT, 5 SQUARE
42-381 100 SHEETS, FELT, 5 SQUARE
42-382 100 SHEETS, FELT, 5 SQUARE
42-383 200 RECYCLED WHITE 5 SQUARE
42-389 200 RECYCLED WHITE 5 SQUARE
Mead U.S.A.



$$1) \quad T_x = T_1 \cos^2 \theta + T_2 \sin^2 \theta - 2T_{12} \sin \theta \cos \theta$$

$$2) \quad T_y = T_1 \sin^2 \theta + T_2 \cos^2 \theta + 2T_{12} \sin \theta \cos \theta$$

$$3) \quad 2 = \frac{T_y}{T_x} = \frac{T_1 \sin^2 \theta + T_2 \cos^2 \theta + 2T_{12} \sin \theta \cos \theta}{T_1 \cos^2 \theta + T_2 \sin^2 \theta - 2T_{12} \sin \theta \cos \theta}$$

$$4) \quad T_{xy} = 0 = T_1 \cos \theta \sin \theta - T_2 \cos \theta \sin \theta + T_{12} (\cos^2 \theta - \sin^2 \theta)$$

What if we do not assume $T_1 \gg T_2$

- From (4)

$$5) \quad T_1 = \frac{T_2 \cos \theta \sin \theta - T_{12} (\cos^2 \theta - \sin^2 \theta)}{\cos \theta \sin \theta}$$

- Equation (5) into (1)

$$T_x = \left[\frac{T_2 \cos \theta \sin \theta - T_{12} (\cos^2 \theta - \sin^2 \theta)}{\cos \theta \sin \theta} \right] \cos^2 \theta$$

$$+ T_2 \sin^2 \theta - 2T_{12} \sin \theta \cos \theta$$

$$T_x = T_2 \cos^2 \theta - T_{12} (\cos^2 \theta - \sin^2 \theta) \cot \theta + T_2 \sin^2 \theta$$

$$- 2T_{12} \sin \theta \cos \theta$$

$$T_x = T_2 (\cos^2 \theta + \sin^2 \theta) - T_{12} (\cos^2 \theta - \sin^2 \theta) \cot \theta$$

$$- 2T_{12} \sin \theta \cos \theta$$

$$6a) \quad T_2 = T_x + T_{12} (\cos^2 \theta - \sin^2 \theta) \cot \theta + 2T_{12} \sin \theta \cos \theta$$

- Equation (6a) into (5)

$$T_1 = \left[T_x + T_{12} (\cos^2 \theta - \sin^2 \theta) \cot \theta + 2T_{12} \sin \theta \cos \theta \right] \frac{\cos \theta \sin \theta}{\cos \theta \sin \theta}$$

$$- T_{12} \frac{(\cos^2 \theta - \sin^2 \theta)}{\cos \theta \sin \theta}$$

$$(6b) \quad T_1 = T_x + T_{12} (\cos^2 \theta - \sin^2 \theta) \cot \theta + 2 T_{12} \sin \theta \cos \theta$$

$$- T_{12} \frac{(\cos^2 \theta - \sin^2 \theta)}{\cos \theta \sin \theta}$$

Note: For plane stress, the sum of the normal stresses exerted on a cubic element of material is independent of the orientation of that element. (p 343 [2])

Thus

$$\nabla_x + \nabla_y = \nabla_1 + \nabla_2$$

$$\begin{aligned} \tau_x + \tau_y &= 2\tau_x + 2\tau_{12}(\cos^2\theta - \sin^2\theta) \cot\theta + 4\tau_{12} \sin\theta \cos\theta \\ &= \tau_{12} \frac{(\cos^2\theta - \sin^2\theta)}{\cos\theta \sin\theta} \end{aligned}$$

$$\nabla_y - \nabla_x = T_{12} \left[2(\cos^2 \theta - \sin^2 \theta) \cot \theta + 4 \sin \theta \cos \theta - \frac{(\cos^2 \theta - \sin^2 \theta)}{\cos \theta \sin \theta} \right]$$

$$7) \quad I_{12} = (\tau_y - \tau_x) \left[2(\cos^2\theta - \sin^2\theta) \cot\theta + 4 \sin\theta \cos\theta - \frac{(\cos^2\theta - \sin^2\theta)}{\cos\theta \sin\theta} \right]$$

$$\nabla_x = \frac{pr}{2t_t} \quad \nabla_y = \frac{pr}{t_t}$$

t_t = Total laminate thickness = $n t$

t = Lamina thickness

n = Number of layers

$$T_X = \frac{pr}{2nt} \quad T_Y = \frac{pr}{nt}$$

Unknown: n, θ

Solution Process

a) Pick an arbitrary n, θ

b) Calculate $\bar{J}_{12} \rightarrow$ Equation (7)

$$\bar{J}_1 \rightarrow (6b)$$

$$\bar{J}_2 \rightarrow (6a)$$

c) Calculate $\bar{J}_y \rightarrow (2)$

$$\bar{J}_x \rightarrow (1)$$

d) Does $\frac{\bar{J}_y}{\bar{J}_x} = 2$

- If not, iterate until it does by changing θ .

13-782
500 SHEETS FILLER 5 SQUARE
42-382 100 SHEETS 5 Y.E. 5 SQUARE
42-382 200 SHEETS 5 Y.E. 5 SQUARE
42-392 100 RECYCLED WHITE 5 SQUARE
42-392 200 RECYCLED WHITE 5 SQUARE
Made in U.S.A.



Finding the number of lamina for a composite pressure vessel

ORIENTED (Cylindrical with spherical ends)

Given: Units

$S_L(+)$	2.85E+05 psi	Longitudinal strength in tension
$S_L(-)$	2.45E+05 psi	Longitudinal strength in compression
$S_T(+)$	7.65E+03 psi	Transverse strength in tension
$S_T(-)$	7.65E+03 psi	Transverse strength in compression
S_{LT}	1.50E+04	In plane shear strength
p	1000 psi	Operating pressure
F.S.	2	Factor of safety
t	0.005 in	Lamina thickness
r	4 in	Radius of pressure vessel

Find:

n **10** Number of layers per orientation angle

1) Obtain F's

F_1	-5.73E-07 (in^2/lbs)
F_{11}	1.43E-11 (in^4/lbs^2)
F_2	0.00E+00 (in^2/lbs)
F_{22}	1.71E-08 (in^4/lbs^2)
F_{66}	4.44E-09 (in^4/lbs^2)
F_{12}	-2.473E-10 (in^4/lbs^2)

2) Get the hoop and longitudinal stresses

σ_l	80000 psi
σ_h	160000 psi

3) Get the principal stresses, and orientation angle. (Assume $\theta = \text{optimal}$)

a) Guess θ

I used "Goal Seek" to set the ratio of the hoop stress and longitudinal stress to 2. This changed the optimal angle from 55 to 45 degrees.

θ	45.0032043	-45.003204 degrees
	0.78545409	-0.7854541 radians
τ_{12}	39998.51	-39998.51 psi
σ_1	120004.47	120004.47 psi
σ_2	119995.53	119995.53 psi
σ_h/σ_l	2.00	2.00

Total stress (psi)

τ_{12} 0.00

σ_2 240008.95

σ_1 239991.05

ORIENTED (Cylindrical with spherical ends)

4) Evaluate Failure Criterion

a) Maximum Stress Criterion:

	psi	σ_1 (psi)	Oriented	Pass Test?
-SL(-)	-2.45E+05	2.40E+05	TRUE	
SL(+)	2.85E+05			
		σ_2 (psi)		
-ST(-)	-7.65E+03	2.40E+05	FALSE	
ST(+)	7.65E+03			
		$ \tau_{12} $ (psi)		
SLT	1.50E+04	0.00E+00	TRUE	

b) von Mises Stress Criterion:

vM	Pass Test?
8.20E-01	TRUE

c) Tsai-Hill Stress Criterion:

T-H	Pass Test?
9.84E+02	FALSE

d) Tsai-Wu Stress Criterion:

T-W	Pass Test?
9.57E+02	FALSE

Finding the number of lamina for a composite pressure vessel

0 DEGREES (Cylindrical with spherical ends)

Given:

	Units	
$S_{L(+)}$	2.85E+05 psi	Longitudinal strength in tension
$S_{L(-)}$	2.45E+05 psi	Longitudinal strength in compression
$S_{T(+)}$	7.65E+03 psi	Traverse strength in tension
$S_{T(-)}$	7.65E+03 psi	Traverse strength in compression
S_{LT}	1.50E+04	In plane shear strength
p	1000 psi	Operating pressure
F.S.	2	Factor of safety
t	0.005 in	Lamina thickness
r	4 in	Radius of pressure vessel

Find:

n **6** **Number of layers**

1) Obtain F's

F_1	-5.73E-07 (in^2/lbs)
F_{11}	1.43E-11 (in^4/lbs^2)
F_2	0.00E+00 (in^2/lbs)
F_{22}	1.71E-08 (in^4/lbs^2)
F_{66}	4.44E-09 (in^4/lbs^2)
F_{12}	-2.473E-10 (in^4/lbs^2)

2) Get the hoop and longitudinal stresses

σ_l	133333.333 psi
σ_h	266666.667 psi

3) Get the principal stresses, and orientation angle. (Assume $\theta = 0$)

σ_1	133333.33 psi
σ_2	266666.67 psi
τ_{12}	0.00 psi

σ_h/σ_l **2.00**

0 DEGREES

(Cylindrical with spherical ends)

4) Evaluate Failure Criterion

a) Maximum Stress Criterion:

	psi	σ_1 (psi)	Pass Test?
-SL(-)	-2.45E+05	1.33E+05	TRUE
SL(+)	2.85E+05		
		σ_2 (psi)	
-ST(-)	-7.65E+03	2.67E+05	FALSE
ST(+)	7.65E+03		
		$ \tau_{12} $ (psi)	
SLT	1.50E+04	0.00E+00	TRUE

b) von Mises Stress Criterion:

vM	Pass Test?
7.59E-01	TRUE

c) Tsai-Hill Stress Criterion:

T-H	Pass Test?
1.21E+03	FALSE

d) Tsai-Wu Stress Criterion:

T-W	Pass Test?
1.20E+03	FALSE

Finding the number of lamina for a composite pressure vessel

90 DEGREES (Cylindrical with spherical ends)

Given:

	Units	
SL(+)	2.85E+05 psi	Longitudinal strength in tension
SL(-)	2.45E+05 psi	Longitudinal strength in compression
ST(+)	7.65E+03 psi	Traverse strength in tension
ST(-)	7.65E+03 psi	Traverse strength in compression
SLT	1.50E+04	In plane shear strength
p	1000 psi	Operating pressure
F.S.	2	Factor of safety
t	0.005 in	Lamina thickness
r	4 in	Radius of pressure vessel

Find:

n	6	Number of layers
----------	----------	-------------------------

1) Obtain F's

F ₁	-5.73E-07 (in ² /lbs)
F ₁₁	1.43E-11 (in ⁴ /lbs ²)
F ₂	0.00E+00 (in ² /lbs)
F ₂₂	1.71E-08 (in ⁴ /lbs ²)
F ₆₆	4.44E-09 (in ⁴ /lbs ²)
F ₁₂	-2.473E-10 (in ⁴ /lbs ²)

2) Get the hoop and longitudinal stresses

σ_l	133333.333 psi
σ_h	266666.667 psi

3) Get the principal stresses, and orientation angle. (Assume $\theta = 0$)

σ_1	266666.67 psi
σ_2	133333.33 psi
τ_{12}	0.00 psi

σ_l/σ_h	2.00
---------------------	------

90 DEGREES (Cylindrical with spherical ends)

4) Evaluate Failure Criterion

a) Maximum Stress Criterion:

	psi	σ_1 (psi)	Pass Test?
-SL(-)	-2.45E+05	2.67E+05	TRUE
SL(+)	2.85E+05		
		σ_2 (psi)	
-ST(-)	-7.65E+03	1.33E+05	FALSE
ST(+)	7.65E+03		
		$ \tau_{12} $ (psi)	
SLT	1.50E+04	0.00E+00	TRUE

b) von Mises Stress Criterion:

vM	Pass Test?
7.59E-01	TRUE

c) Tsai-Hill Stress Criterion:

T-H	Pass Test?
3.04E+02	FALSE

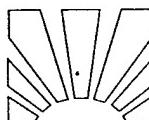
d) Tsai-Wu Stress Criterion:

T-W	Pass Test?
2.87E+02	FALSE

E

n	σ_l	σ_h	For a positive orientation angle				For a negative orientation angle			
			τ_{12}	σ_1	σ_2		τ_{12}	σ_1	σ_2	
1	800000	1600000	399985.05	1200044.74	1199955.26	-399985.05	1200104.31	1200014.84		
2	400000	800000	199992.53	600022.37	599977.63	-199992.53	600052.16	600007.42		
3	266667	533333.333	133328.35	400014.91	399985.09	-133328.35	400034.77	400004.95		
4	200000	400000	99996.26	300011.18	299988.82	-99996.26	300026.08	300003.71		
5	160000	320000	79997.01	240008.95	239991.05	-79997.01	240020.86	240002.97		
6	133333	266666.667	66664.18	200007.46	199992.54	-66664.18	200017.39	200002.47		
7	114286	228571.429	57140.72	171434.96	171422.18	-57140.72	171443.47	171430.69		
8	100000	200000	49998.13	150005.59	149994.41	-49998.13	150013.04	150001.85		
9	88888.9	177777.778	44442.78	133338.30	133328.36	-44442.78	133344.92	133334.98		
10	80000	160000	39998.51	120004.47	119995.53	-39998.51	120010.43	120001.48		
11	72727.3	145454.545	36362.28	109094.98	109086.84	-36362.28	109100.39	109092.26		
12	66666.7	133333.333	33332.09	100003.73	99996.27	-33332.09	100008.69	100001.24		
13	61538.5	123076.923	30768.08	92311.13	92304.25	-30768.08	92315.72	92308.83		
14	57142.9	114285.714	28570.36	85717.48	85711.09	-28570.36	85721.74	85715.35		
15	53333.3	106666.667	26665.67	80002.98	79997.02	-26665.67	80006.95	80000.99		
16	50000	100000	24999.07	75002.80	74997.20	-24999.07	75006.52	75000.93		
17	47058.8	94117.6471	23528.53	70590.87	70585.60	-23528.53	70594.37	70589.11		
18	44444.4	88888.8889	22221.39	66669.15	66664.18	-22221.39	66672.46	66667.49		
19	42105.3	84210.5263	21051.84	63160.25	63155.54	-21051.84	63163.38	63158.68		
20	40000	80000	19999.25	60002.24	59997.76	-19999.25	60005.22	60000.74		
21	38095.2	76190.4762	19046.91	57144.99	57140.73	-19046.91	57147.82	57143.56		
22	36363.6	72727.2727	18181.14	54547.49	54543.42	-18181.14	54550.20	54546.13		
23	34782.6	69565.2174	17390.65	52175.86	52171.97	-17390.65	52178.45	52174.56		
24	33333.3	66666.6667	16666.04	50001.86	49998.14	-16666.04	50004.35	50000.62		
25	32000	64000	15999.40	48001.79	47998.21	-15999.40	48004.17	48000.59		
26	30769.2	61538.4615	15384.04	46155.57	46152.13	-15384.04	46157.86	46154.42		
27	29629.6	59259.2593	14814.26	44446.10	44442.79	-14814.26	44448.31	44444.99		
28	28571.4	57142.8571	14285.18	42858.74	42855.55	-14285.18	42860.87	42857.67		
29	27586.2	55172.4138	13792.59	41380.85	41377.77	-13792.59	41382.91	41379.82		
30	26666.7	53333.3333	13332.84	40001.49	39998.51	-13332.84	40003.48	40000.49		
31	25806.5	51612.9032	12902.74	38711.12	38708.23	-12902.74	38713.04	38710.16		
32	25000	50000	12499.53	37501.40	37498.60	-12499.53	37503.26	37500.46		
33	24242.4	48484.8485	12120.76	36364.99	36362.28	-12120.76	36366.80	36364.09		
34	23529.4	47058.8235	11764.27	35295.43	35292.80	-11764.27	35297.19	35294.55		
35	22857.1	45714.2857	11428.14	34286.99	34284.44	-11428.14	34288.69	34286.14		
36	22222.2	44444.4444	11110.70	33334.58	33332.09	-11110.70	33336.23	33333.75		
37	21621.6	43243.2432	10810.41	32433.64	32431.22	-10810.41	32435.25	32432.83		
38	21052.6	42105.2632	10525.92	31580.12	31577.77	-10525.92	31581.69	31579.34		
39	20512.8	41025.641	10256.03	30770.38	30768.08	-10256.03	30771.91	30769.61		
40	20000	40000	9999.63	30001.12	29998.88	-9999.63	30002.61	30000.37		
41	19512.2	39024.3902	9755.73	29269.38	29267.20	-9755.73	29270.84	29268.65		
42	19047.6	38095.2381	9523.45	28572.49	28570.36	-9523.45	28573.91	28571.78		
43	18604.7	37209.3023	9301.98	27908.02	27905.94	-9301.98	27909.40	27907.32		
44	18181.8	36363.6364	9090.57	27273.74	27271.71	-9090.57	27275.10	27273.06		
45	17777.8	35555.5556	8888.56	26667.66	26665.67	-8888.56	26668.98	26667.00		

Total Stress for oriented plies			For 0 degrees		For 90 degrees	
τ_{12T}	σ_{1T}	σ_{2T}	σ_2	σ_1	σ_2	σ_1
0.00	2400149.05	2399970.10	1600000	800000	800000	1600000
0.00	1200074.53	1199985.05	800000	400000	400000	800000
0.00	800049.68	799990.03	533333.3	266666.7	266667	533333
0.00	600037.26	599992.52	400000	200000	200000	400000
0.00	480029.81	479994.02	320000	160000	160000	320000
0.00	400024.84	399995.02	266666.7	133333.3	133333	266667
0.00	342878.44	342852.87	228571.4	114285.7	114286	228571
0.00	300018.63	299996.26	200000	100000	100000	200000
0.00	266683.23	266663.34	177777.8	88888.89	88888.9	177778
0.00	240014.91	239997.01	160000	80000	80000	160000
0.00	218195.37	218179.10	145454.5	72727.27	72727.3	145455
0.00	200012.42	199997.51	133333.3	66666.67	66666.7	133333
0.00	184626.85	184613.08	123076.9	61538.46	61538.5	123077
0.00	171439.22	171426.44	114285.7	57142.86	57142.9	114286
0.00	160009.94	159998.01	106666.7	53333.33	53333.3	106667
0.00	150009.32	149998.13	100000	50000	50000	100000
0.00	141185.24	141174.71	94117.65	47058.82	47058.8	94117.6
0.00	133341.61	133331.67	88888.89	44444.44	44444.4	88888.9
0.00	126323.63	126314.22	84210.53	42105.26	42105.3	84210.5
0.00	120007.45	119998.50	80000	40000	40000	80000
0.00	114292.81	114284.29	76190.48	38095.24	38095.2	76190.5
0.00	109097.68	109089.55	72727.27	36363.64	36363.6	72727.3
0.00	104354.31	104346.53	69565.22	34782.61	34782.6	69565.2
0.00	100006.21	99998.75	66666.67	33333.33	33333.3	66666.7
0.00	96005.96	95998.80	64000	32000	32000	64000
0.00	92313.43	92306.54	61538.46	30769.23	30769.2	61538.5
0.00	88894.41	88887.78	59259.26	29629.63	29629.6	59259.3
0.00	85719.61	85713.22	57142.86	28571.43	28571.4	57142.9
0.00	82763.76	82757.59	55172.41	27586.21	27586.2	55172.4
0.00	80004.97	79999.00	53333.33	26666.67	26666.7	53333.3
0.00	77424.16	77418.39	51612.9	25806.45	25806.5	51612.9
0.00	75004.66	74999.07	50000	25000	25000	50000
0.00	72731.79	72726.37	48484.85	24242.42	24242.4	48484.8
0.00	70592.62	70587.36	47058.82	23529.41	23529.4	47058.8
0.00	68575.69	68570.57	45714.29	22857.14	22857.1	45714.3
0.00	66670.81	66665.84	44444.44	22222.22	22222.2	44444.4
0.00	64868.89	64864.06	43243.24	21621.62	21621.6	43243.2
0.00	63161.82	63157.11	42105.26	21052.63	21052.6	42105.3
0.00	61542.28	61537.69	41025.64	20512.82	20512.8	41025.6
0.00	60003.73	59999.25	40000	20000	20000	40000
0.00	58540.22	58535.86	39024.39	19512.2	19512.2	39024.4
0.00	57146.41	57142.15	38095.24	19047.62	19047.6	38095.2
0.00	55817.42	55813.26	37209.3	18604.65	18604.7	37209.3
0.00	54548.84	54544.77	36363.64	18181.82	18181.8	36363.6
0.00	53336.65	53332.67	35555.56	17777.78	17777.8	35555.6



February 3, 1998

C98-7001-1A

COMPOSITES HORIZONS, INC.

Lt. Andrew Martin
Air Force Academy
4715 Garden Ranch Road #308
Colorado Springs, CO 80918

Reference: CARE-MOLD™ Material

Dear Lt. Martin:

Thank you for your interest in the CARE-MOLD™ CAstable and REmovable mandrel system. CARE-MOLD™ mandrels are currently used in various production and prototype applications where hard tooling is either design or cost prohibitive.

CARE-MOLD dry powder is supplied in 50 lb plastic containers, mixed with water and cast into any closed mold to form a net shape mandrel. The excellent casting and thermal properties of CARE-MOLD eliminate the need for corrections in the mold for shrinkage or thermal expansion. Because CARE-MOLD is cast at room temperature, low temperature tooling materials may be used. Once the CARE-MOLD is cast it can then be removed from the mold in one (1) hour and dried at its end use temperature, (minimum 350° to maximum 800°F). The CARE-MOLD must then be sealed to assure a smooth tool surface. The choice of which sealing agent will depend on the end use temperature of the CARE-MOLD. CHI can recommend a variety of sealing agents specific to the configuration and use of the mandrel. Once sealed and released the CARE-MOLD can be used to form ribs, hats, or tubes. CARE-MOLD has been used in compression molding, filament winding, and hand lay-up of various polyimide, thermoplastic, and epoxy parts. CARE-MOLD can be used at 500 psi and 800°F.

Pricing for the CARE-MOLD™ is:

50 lb	\$5.00/lb
200 lb	\$4.50/lb
500 lb	\$4.25/lb
1000 lb	\$4.00/lb

Terms: Net 30 days, F.O.B. Covina

Availability: 5 working days

If you have technical questions or are interested in placing an order for the CARE-MOLD material contact the undersigned at (818) 331-0861. We are anxious to work with you on programs that could benefit from the CARE-MOLD process and look forward to servicing your requirements:

Very truly yours,

Jeffrey T. Hynes (Arura)
Vice President Marketing & Sales

Encl: MSDS Sheets
CARE-MOLD™ Data Sheets
CARE-MOLD™ Instructions

60

INTRODUCING —

CARE-MOLD Process

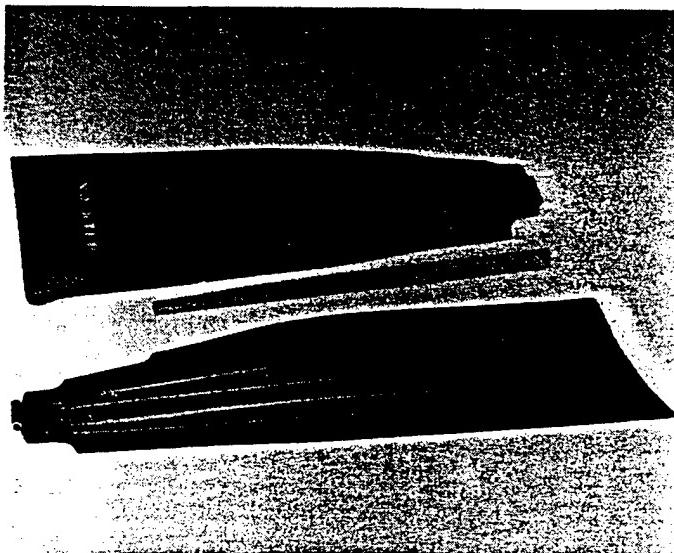
Using our CAstable and REmovable wash-out mandrel, CHI can provide high-temperature processing of:

PMR-15, PMR-II-700

APC - 2*, APC-HTX*, APC-HTA,
PAS-2*, RADEL* C, AVIMID* N,
AVIMID* K-III, CYPAC* 7005,
CYPAC* 7156, and many others.

* Registered Tradenames

- Temperatures to 800 F
- Pressures to 400 psi
- Helps eliminate secondary bonding
- Enables complex molding



Co-Cured PMR-15 Application
Uses CARE-MOLD Process to
Form Closed-End Hollow Rib

Composites Horizons, Inc. has developed a proprietary expendable mandrel material and process for use at the high temperatures and pressures required for these new space-age materials. Identified as the CARE-MOLD Process, it often permits the co-curing or co-consolidation necessary to avoid difficult and suspect secondary bonding operations. The CARE-MOLD Process permits casting at room temperature, and it can be used for high temperature breakaway tooling of complex stiffeners and shapes.

Preliminary tests of the molding material used in the CARE-MOLD Process reveal the following properties:

PROPERTY	TEMPERATURE		
	ROOM	600 F	800 F
Compressive Strength (psi)	290	450	490
Young's Modulus (psi)	50,000	38,000	38,000

In the range of 120 to 660 F, the material has an effective Coefficient of Thermal Expansion (CTE) of 0.00000146 in/in/F. At temperatures above 300 F however, the slope of the CTE curve is 0.00000185 in/in/F.

To see if the CARE-MOLD Process can solve a high temperature processing problem for you, please contact Tom Hynes, President, or Milt Anderson, Vice-President of Engineering at (818) 331-0861 or by FAX at (818) 339-3220

Building on the Past

. Growing toward the Future

CARE-MOLD 800

CASTABLE AND REMOVEABLE MANDREL MATERIAL

DIRECTIONS FOR USE:

- MIXING:** Mix 40 grams of water per 100 grams of dry powder mix. Add appropriate water amount to a blender and slowly add powder while blending at a high speed. After all the powder has been added, continue mixing for 30 seconds to assure complete mixture. Mold must be poured within 8-10 minutes after introduction of water.
- TOOLING:** CARE-MOLD 800 can be cast into any metallic or plastic mold. Shrinkage and expansion is minimal so molds can be made to net size. Tools should be sealed and released prior to use. Waxes or spray-on releases work well, but motor oil works best on all types of tooling.
- CURE:** Molds should be allowed to set for a minimum of 45 minutes before de-molding. Molds can then be introduced to a drying oven at 500 to 600 °F. Molds should be dried for a minimum of 4 hours at temperature.
- SEALING:** Molds can be spot repaired if necessary by adding a small amount of mixed powder and dried with a heat gun. Molds should be sealed with Teflon tape or other sealant to prevent resin bleed into the mandrel.
- WASH-OUT:** CARE-MOLD 800 is washed-out with pressured water and agitation. Drilling or casting removable rods into the mandrel will facilitate in the washing out of the mold.
- SHELF LIFE:** One year from date of shipment. Once container is opened, contents may deteriorate on exposure to moisture.
- WARNING:** California Prop. 65 This product contains crystalline silica, a chemical known to the State of California to cause cancer.
- WARRANTY:** Composites Horizons, Inc. (CHI) warrants to the Buyer, its employees and agents that CARE-MOLD 800 will be free of defects in materials and workmanship for one year from date of shipment, subject to CHI's Standard Terms and Conditions of Sale, dated May 1995, which excludes liability for consequential loss or damage sustained directly or indirectly as a result of defect or misuse.

COMPOSITES HORIZONS, INC.

1471 INDUSTRIAL PARK STREET
COVINA, CA 91722-3499
PHONE: (626) 331-0861
FAX: (626) 339-3220

BATCH NO. _____

D.O.S. _____

Net Weight 50 lb (22.7 kg)

FOR INDUSTRIAL USE ONLY

PROPRIETARY MATERIAL - PROTECTED BY U.S. PATENT

MATERIAL SAFETY DATA SHEET

Composites Horizons, Inc. (CHI)
1471 Industrial Park Street
Covina, CA 91722-3499
Emergency Phone: (626) 331-0861

CARE-MOLD™ 800
ISSUE DATE: November 27, 1989
REVISION DATE: August 8, 1997

HAZARDOUS INGREDIENTS

INGREDIENT	PERCENTAGE
CRISTOBALITE	30-35
COMMON NAMES AND SYNONYMS SILICA	CAS NUMBER 14464-46-1
PEL 0.05mg/m³ (Respirable dust)	UN NUMBER Not Applicable
TLV 0.05mg/m³ (Respirable dust)	

INGREDIENT	PERCENTAGE
QUARTZ	30-35
COMMON NAMES AND SYNONYMS SILICA	CAS NUMBER 14808-60-7
PEL 0.1mg/m³ (Respirable dust)	UN NUMBER Not Applicable
TLV 0.1mg/m³ (Respirable dust)	

INGREDIENT	PERCENTAGE
CALCIUM SULFATE HEMIHYDRATE	25-30
COMMON NAMES AND SYNONYMS GYPSUM	CAS NUMBER 7778-18-9
PEL 5mg/m³ (Respirable fraction)	UN NUMBER Not Applicable
TLV 10mg/m³	

PHYSICAL/CHEMICAL CHARACTERISTICS

BOILING POINT Not Applicable	SPECIFIC GRAVITY Not Established	VAPOR PRESSURE Not Applicable	MELTING POINT Not Applicable
VAPOR DENSITY Not Applicable	EVAPORATION RATE Not Applicable	SOLUBILITY IN WATER Slightly Soluble	
APPEARANCE AND ODOR Light Grey Odorless Powder			

FIRE AND EXPLOSION HAZARD DATA

FLASH POINT Not Applicable	LIMITS Lel: Not Applicable Uel: Not Applicable
EXTINGUISHING MEDIA Not Applicable	
SPECIAL FIREFIGHTING PROCEDURES Not Applicable	
UNUSUAL FIRE AND EXPLOSION HAZARDS Not Applicable	

REACTIVITY DATA

STABILITY Stable
CONDITIONS TO AVOID None
INCOMPATIBILITY (MATERIALS TO AVOID) Hydrofluoric acid, in which silica will dissolve and produce the corrosive gas, silicon tetrafluoride.
HAZARDOUS DECOMPOSITION PRODUCTS Not Applicable
HAZARDOUS POLYMERIZATION Will not occur
CONDITIONS TO AVOID None

HEALTH HAZARD DATA

CARCINOGENICITY Yes	NTP? No	IARC? Yes	OSHA REGULATED? Yes
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EFFECTS AND HAZARDS OF OVEREXPOSURE (ACUTE AND CHRONIC)

EFFECTS AND HAZARDS OF EYE CONTACT May cause irritation.
EFFECTS AND HAZARDS OF SKIN CONTACT May cause dry feeling on skin.
EFFECTS AND HAZARDS OF INHALATION (BREATHING) Prolonged exposure to respirable crystalline silica may cause chronic lung injury (silicosis). Acute developing silicosis may occur in a short time in heavy exposure. Silicosis is a form of disabling pulmonary fibrosis which can be progressive and may lead to death. The International Agency for Research on Cancer (IRAC) reports limited evidence of the carcinogenicity of crystalline silica to humans. IRAC class 2A.
Composites Horizons, Inc. Accepts no responsibility and disclaims all liability for harmful health effects. Customers must comply with all applicable health and safety regulations relating to the safe handling of our silica containing products.
EFFECTS AND HAZARDS OF INGESTION (SWALLOWING) Hardens when wetted and if ingested may result in an obstruction.

EMERGENCY AND FIRST AID PROCEDURES

TREATMENT FOR EYE CONTACT

Wash eyes immediately with large amounts of water, lifting lower and upper lids occasionally. If irritation persists, get medical attention.

TREATMENT FOR SKIN CONTACT

Wash with soap and water. Use hand lotion.

TREATMENT FOR INHALATION (BREATHING)

Move exposed person to fresh air at once.

TREATMENT FOR INGESTION (SWALLOWING)

Consult physician.

PRECAUTIONS FOR SAFE HANDLING AND USE

STEPS TO BE TAKEN IN CASE MATERIAL IS RELEASED OR SPILLED

Ventilate area of spill or release. Vacuum or sweep up - avoid unnecessary stirring or handling in order to prevent formation of dust.

WASTE DISPOSAL METHOD

Landfill

PRECAUTIONS TO BE TAKEN IN HANDLING AND STORING

Do not breathe dust. Keep container closed.

OTHER PRECAUTIONS

Use according to directions. Follow prescribed mixing procedure.

CONTROL MEASURES

RESPIRATORY PROTECTION

Use NIOSH-approved equipment. Also see ANSI Standard Z88.2-1980, "Practices for Respiratory Protection."

VENTILATION

LOCAL EXHAUST VENTILATION

Use sufficient local exhaust to reduce the level of respirable crystalline silica to the PEL.

SPECIAL VENTILATION

None.

MECHANICAL (GENERAL) VENTILATION

None.

OTHER VENTILATION

None.

PROTECTIVE GLOVES

Gloves optional.

EYE PROTECTION

Eye goggles.

OTHER PROTECTIVE EQUIPMENT

Not Applicable.

WORK AND HYGIENIC PRACTICES

Handle in accordance with good personal hygiene and safety practices. These practices include avoiding unnecessary exposure.

ADDITIONAL INFORMATION

This MSDS was prepared in accordance with the requirements of the OSHA Hazard Communication Standard (29CFR 1910.1200) and is to be used only for this product. The information contained in this sheet is, to the best of our knowledge, believed to be accurate.

STATE RTX**California Prop. 65 Warning:**

This product contains crystalline silica, a chemical known to the State of California to cause cancer.

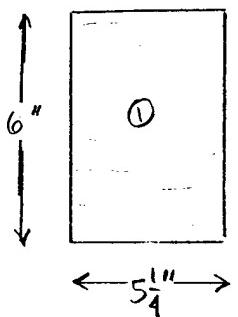
RATINGS	HAZARD INDEX	
HMIS	4	Severe Hazard
H3	3	Serious Hazard
F0	2	Moderate Hazard
R0	1	Slight Hazard
	0	Minimum Hazard

SHIPPING INFORMATION

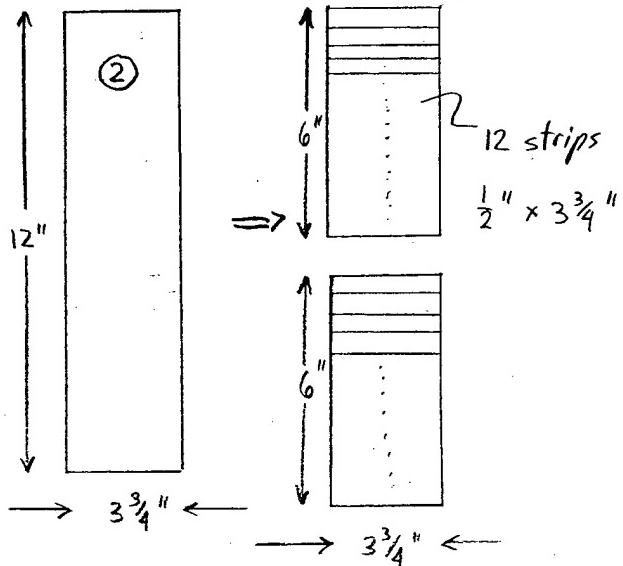
Not regulated.

• 0 Degrees

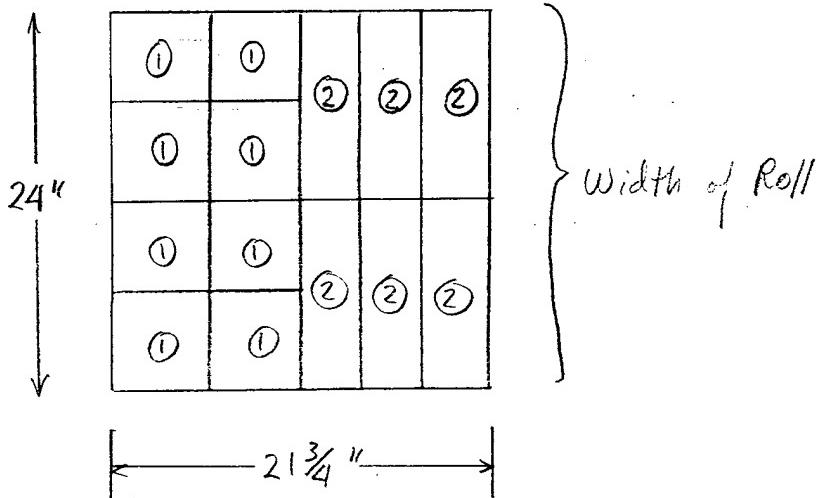
Cylindrical



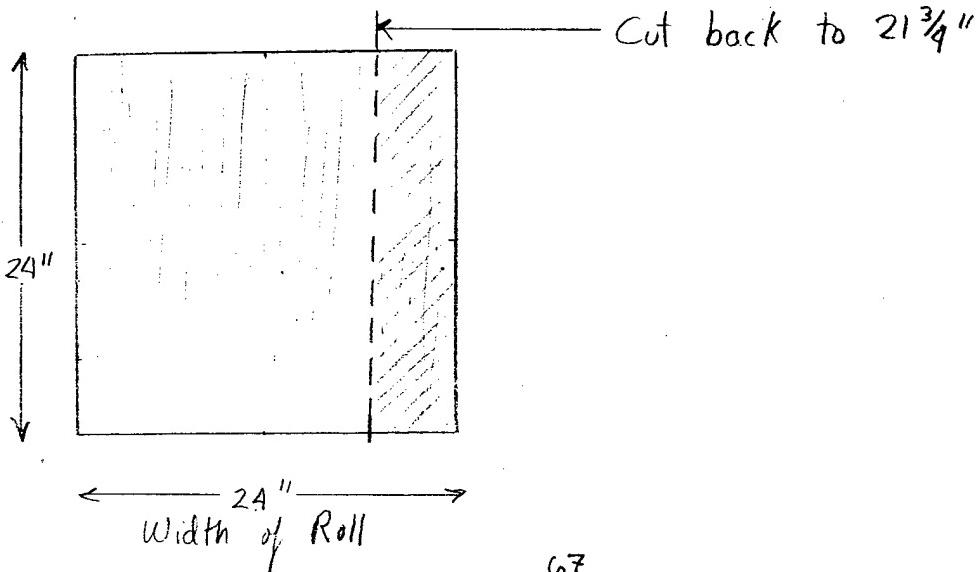
Spherical Strips



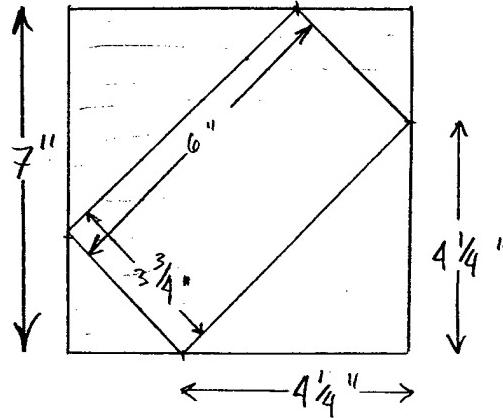
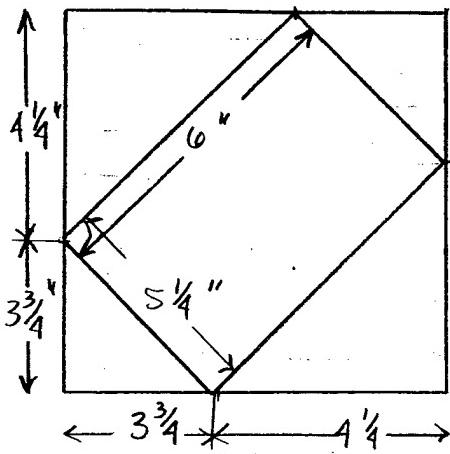
From roll of unitape:



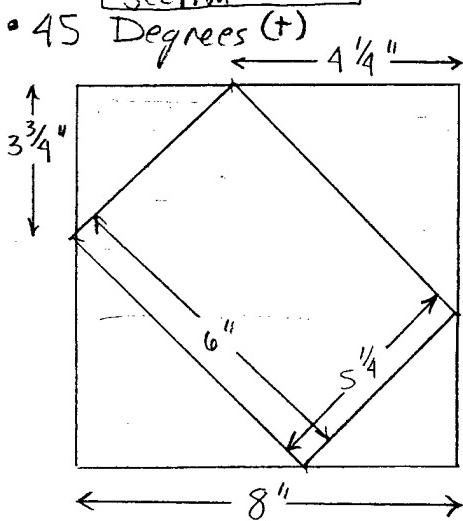
• 90 Degrees



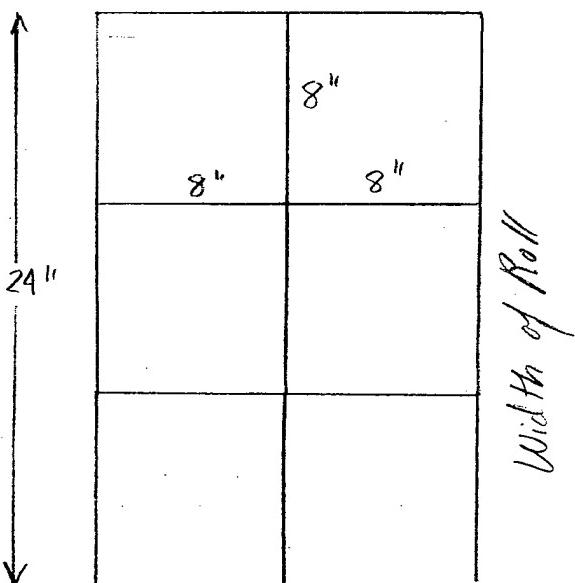
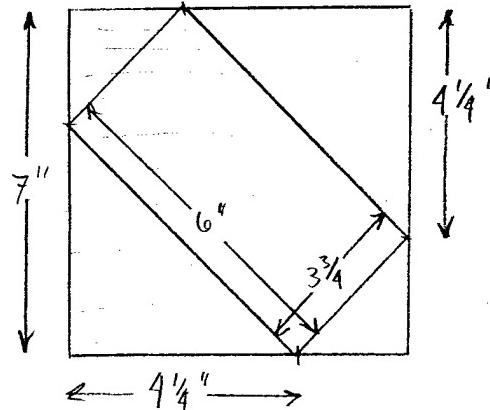
• 45 Degrees (-) (Note this really looks like a +45° ply but when applied to the mold, it reverses to -45° to remove the backing.)



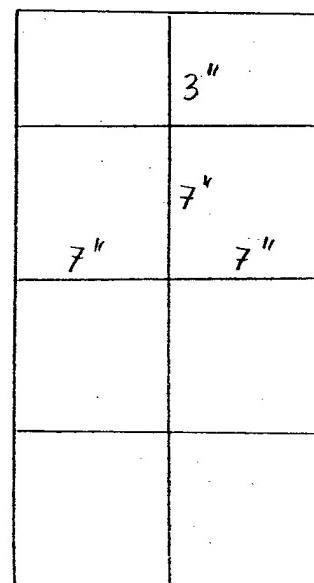
Cylindrical
Section



Spherical Section
Strips ($\frac{1}{2}'' \times 3\frac{3}{4}''$)



Width of Roll



Dimensionless analysis: What if we want to make a prototype. What pressure do we test at for an equivalent pressure if dimensions have changed?

$$\sigma_x = \frac{pr}{2t_1} = \frac{pr}{2nt}$$

$$p = \frac{2nt\sigma_x}{r}$$

Assume the thickness of material remains constant.
(Lamina thickness)

Set

$$\frac{p_1}{\left[\frac{2n_1 t_1 \sigma_{x_1}}{r_1} \right]} = \frac{p_2}{\left[\frac{2n_2 t_2 \sigma_{x_2}}{r_2} \right]}$$

$$p_2 = p_1 \left[\frac{2n_2 t_2 \sigma_{x_2}}{2n_1 t_1 \sigma_{x_1}} \right] \left[\frac{r_1}{r_2} \right]$$

$$\text{If: } t_1 = t_2$$

$n_{1,2} \rightarrow$ arbitrary values, may leave the same or not

$$\sigma_{x_1} = \sigma_{x_2} \rightarrow \text{for simplification}$$

$$p_2 = p_1 \left(\frac{r_1}{r_2} \right)$$